

1. Let $\tan \tan x = 2$. Which is $\cot x$?

Solution.

Firstly, let solve the equation

$$\tan \tan x = 2$$

for $\tan x$.

As $2 > 0$, then we ought to use the following roots of the equation $\tan y = a$:

$$y = \arctan a + \pi n, \quad n \in \mathbb{Z}.$$

So, one can receive:

$$\tan x = \arctan 2 + \pi n, \quad n \in \mathbb{Z}.$$

Now, we shall use the trigonometric identity

$$\tan y \cdot \cot y = 1,$$

which observes for all $y \neq \frac{\pi k}{2}, \quad k \in \mathbb{Z}$.

As $\arctan 2 + \pi n$ does not equal to $\frac{\pi k}{2}$ for any integer n and k , then

$$\cot x = \frac{1}{\tan x} = \frac{1}{\arctan 2 + \pi n}, \quad n \in \mathbb{Z}.$$

Answer: $\cot x = \frac{1}{\arctan 2 + \pi n}, \quad n \in \mathbb{Z}$.