

**Task.** If  $f(x) = f(y)$  does it mean  $x = y$ ?

**Solution.** Thus we have a function  $f : A \rightarrow B$  between two sets  $A$  and  $B$ . Also we have two points  $x, y \in A$  such that  $f(x) = f(y)$ . The question is whether  $x = y$ .

The answer is that “this depends on  $f$ ”. It is possible to construct examples when  $f(x) = f(y)$  implies  $x = y$ , and examples when  $f(x) = f(y)$  does not imply  $x = y$ .

**Example 1:**  $f(x) = f(y)$  **implies**  $x = y$ .

Define a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = 2x$ . Then if  $f(x) = f(y)$  for some  $x, y$ , that is

$$2x = 2y$$

then

$$x = y.$$

**Example 2:**  $f(x) = f(y)$  **does not imply**  $x = y$ .

Define a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x^2$ . Notice that

$$f(1) = f(-1) = 1^2 = (-1)^2 = 1$$

however

$$1 \neq -1.$$