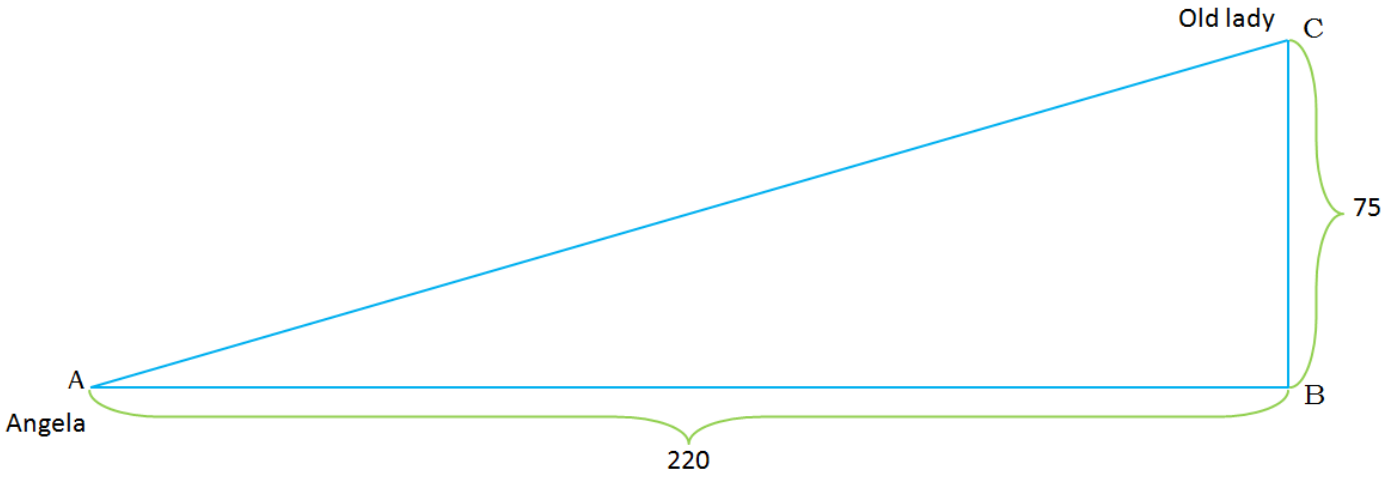
Solution.



1. Angela runs $AB = 220\ m$ and then swims $BC = 75\ m$. Let's calculate how long it takes her:

$$t = \frac{220\ m}{180\ m/min} + \frac{75\ m}{80\ m/min} = \frac{11}{9}\ min + \frac{15}{16}\ min = \frac{311}{144}\ min \text{ or } \boldsymbol{2.16\ min}$$

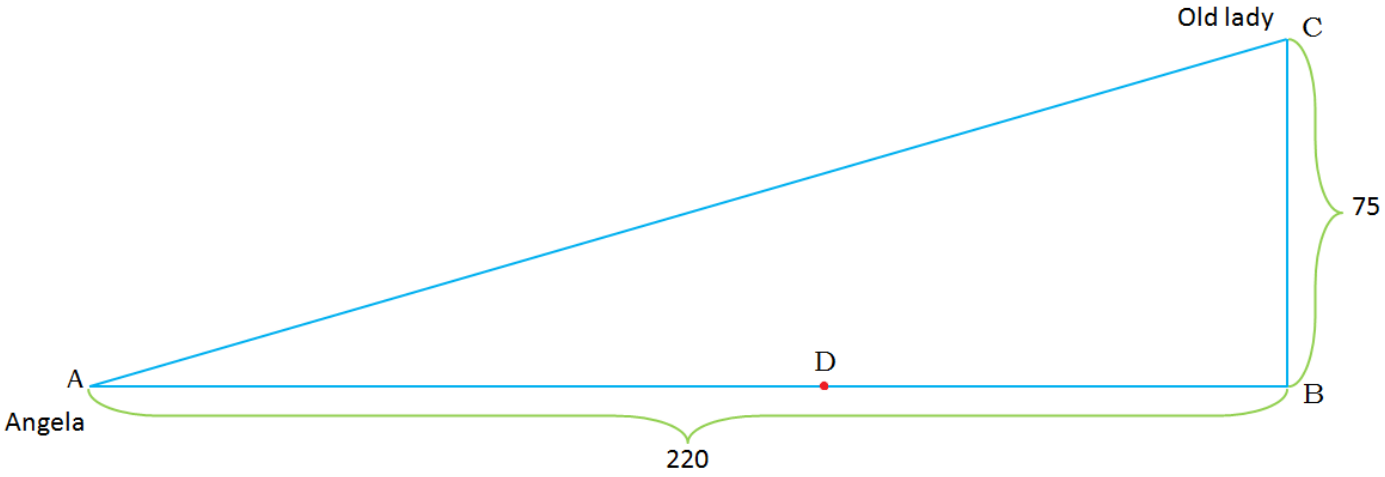
2. She had swum the entire distance $= AC$. So use the Pythagorean theorem to find it:

$$AC = \sqrt{AB^2 + BC^2} = 5\sqrt{2161}\ m \text{ or } 232.43\ m$$

Let's calculate how long it takes her:

$$t = \frac{232.43\ m}{80\ m/min} = \boldsymbol{2.91\ min}$$

3. To find the quickest time we must imagine that at some point D along the stretch of shoreline from A to B Angela stops running and swims directly from D to C:



Let x be the distance that she runs from A to D, $x = AD$, then $BD = 220 - x$. Let's find CD using the Pythagorean theorem:

$$CD = \sqrt{(220 - x)^2 + 75^2} = \sqrt{x^2 - 440x + 54025}$$

So we have a distance from A to C as a function of x:

$$AC = AD + CD = x + \sqrt{x^2 - 440x + 54025}$$

Then calculate how long it takes her:

$$t = \frac{AD}{220} + \frac{CD}{80} = \frac{x}{220} + \frac{\sqrt{x^2 - 440x + 54025}}{80} = f(x)$$

Use calculus to find where this function has a minimum:

$$f'(x) = \frac{1}{220} + \frac{2x - 440}{160\sqrt{x^2 - 440x + 54025}} = 0$$

Solve it for x. Subtract $\frac{1}{220}$ from both sides:

$$\frac{2x - 440}{160\sqrt{x^2 - 440x + 54025}} = -\frac{1}{220}$$

Multiply both sides by 160:

$$\frac{2x - 440}{\sqrt{x^2 - 440x + 54025}} = -\frac{8}{11}$$

Divide both sides by $2x - 440$:

$$\frac{1}{\sqrt{x^2 - 440x + 54025}} = -\frac{4}{11(x - 220)}$$

Take reciprocals of both sides:

$$\sqrt{x^2 - 440x + 54025} = -\frac{11}{4}(x - 220)$$

Raise both sides to the power of two:

$$x^2 - 440x + 54025 = \frac{121}{16}(x - 220)^2$$

Solve the quadratic equation:

$$x = 220 \pm 20\sqrt{\frac{15}{7}}$$

The solution

$$x = 220 + 20\sqrt{\frac{15}{7}}$$

is incorrect.

The solution

$$x = 220 - 20\sqrt{\frac{15}{7}}$$

is correct.

So

$$AD = 220 - 20\sqrt{\frac{15}{7}} \approx 190.72\ m$$

$$BD = 220 - \left(220 - 20\sqrt{\frac{15}{7}}\right)$$

Let's find t:

$$t = \frac{220 - 20\sqrt{\frac{15}{7}}}{220} + \frac{\sqrt{\left(220 - 20\sqrt{\frac{15}{7}}\right)^2 - 440 \cdot \left(220 - 20\sqrt{\frac{15}{7}}\right) + 54025}}{80} = \mathbf{1.87\ (min)}$$

Answer:

1. $t = 2.16\ min;$
2. $t = 2.91\ min;$
3. $t = 1.87\ min.$