According to necessary condition of existence of proper integral

$\left|f\left(x\right)\right| \le C \forall x \in \left[a,b\right]$

So,

$$\lim_{x\to b^-}\int_x^b f\left(t\right)dt \le \lim_{x\to b^-}\int_x^b \left|f\left(t\right)\right|dt \le \lim_{x\to b^-}\int_x^b Cdt = \lim_{x\to b^-} C\left(b-x\right) = 0$$

Having used the property of direction of integration

$$\lim_{x\to b^-}\int_x^b f\left(t\right)dt = -\lim_{x\to b^-}\int_b^x f\left(t\right)dt \ge -\lim_{x\to b^-}\int_b^x \left|f\left(t\right)\right|dt \ge -\lim_{x\to b^-}\int_b^x Cdt = -\lim_{x\to b^-} C(x-b) = 0$$

Finally we have

$$\begin{cases} \lim\limits_{x\to b^-}\int_x^b f\left(t\right)dt \le 0 \\ \lim\limits_{x\to b^-}\int_x^b f\left(t\right)dt \ge 0 \end{cases} \Rightarrow \lim_{x\to b^-}\int_x^b f\left(t\right)dt = 0$$