Find the curve y=f(x) in the xy-plane that has the given properties. f(x) has a slope at each point given by -(1/x^2) and passes through the point (1/4,6)

Solution:

We have the differential equation:

$$\frac{dy}{dx} = \frac{1}{x^2} =>$$

$$y = -\frac{1}{x} + C \ (1), \qquad C - constant$$

Solve the Cauchy problem: find the specification of the solution of the differential equation, given that the curve goes through the point (0.25, 6):

$$x = 0.25, y = 6 => (1):$$

$$6 = -\frac{1}{0.25} + C$$

$$6 = -4 + C$$

$$C = 10$$

Substitute the constant in the equation (1):

$$y = -\frac{1}{x} + 10$$

Answer: $f(x) = -\frac{1}{x} + 10$