Maybe you want to understand how to solve systems of differential equations using matrices.

Let we have the system of differential equations $Y' = AY$. These steps help you to solve this system:

1. Find the matrix P that diagonalises A, i.e. the matrix P such that $D = P^{-1}AP$, where D is a diagonal matrix.
2. Make the change of variable $Y = PU, Y' = PU'$. We then have

$$Y' = AY \Leftrightarrow PU' = APU \Leftrightarrow U' = P^{-1}APU \Leftrightarrow U' = DU$$

3. Solve the system $U' = DU$ for U.
4. The solution to the system of differential equation is $Y = PU$, where U is the solution to $U' = DU$.

Let's see how it works. First we have to find the matrix P, i.e. we have to find **the eigenvalues and eigenvectors** of the matrix A. Let we have the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$$

Then

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = \lambda^2 + \lambda - 6 = (\lambda - 2)(\lambda + 3) = 0$$

so that A has two eigenvalues: $\lambda = 2$ and $\lambda = -3$.

Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$$AX = 2X \Leftrightarrow \begin{cases} -x + y = 0 \\ 4x - 4y = 0 \end{cases} \Leftrightarrow x = y \Leftrightarrow X = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$AX = -3X \Leftrightarrow \begin{cases} 4x + y = 0 \\ 4x - 4y = 0 \end{cases} \Leftrightarrow y = -4x \Leftrightarrow X = \beta \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

The matrix P is therefore $P = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}$ and the diagonal matrix D is $D = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$

Let $U = \begin{pmatrix} u_1(x) \\ u_2(x) \end{pmatrix}$. The matrix equality $U' = DU$ is equivalent to

$$\begin{cases} u_1'(x) = 2u_1(x) \\ u_2'(x) = -3u_2(x) \end{cases},$$

which is easily solved:

$$\begin{cases} u_1'(x) = 2u_1(x) \\ u_2'(x) = -3u_2(x) \end{cases} \Leftrightarrow u_1(x) = Ae^{2x}, u_2(x) = Be^{-3x} \Leftrightarrow U = \begin{pmatrix} Ae^{2x} \\ Be^{-3x} \end{pmatrix}$$

$Y = PU$ is the solution to $Y' = AY$ and we have

$$Y = PU = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} Ae^{2x} \\ Be^{-3x} \end{pmatrix} = \begin{pmatrix} Ae^{2x} + Be^{-3x} \\ Ae^{2x} - 4Be^{-3x} \end{pmatrix}$$

If you are also given initial conditions, i.e. conditions of the form

$$Y(0) = \begin{pmatrix} u(0) \\ v(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

you can find the value of A and the value of B:

$$Y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Leftrightarrow \begin{cases} A + B = 1 \\ A - 4B = -1 \end{cases} \Leftrightarrow \begin{cases} A = \frac{3}{5} \\ B = \frac{2}{5} \end{cases},$$

and finally the solution to the initial value problem is $Y = \frac{1}{5}\begin{pmatrix} 3e^{2x} + 2e^{-3x} \\ 3e^{2x} - 8e^{-3x} \end{pmatrix}$