33562:

Task. Let sin m = .11. Which of the following is sin m/2?

A. 0.79

B. 0.78

C. 0.77

D. 0.05

Solution.

Solution to the equation $\sin(m) = 0.11$ is
$$m = (-1)^k \sin^{-1}(0.11) + k\pi \approx (-1)^k 0.110223 + k\pi \,, \tag{1}$$
where $\pi \approx 3.14159$, k is integer.

Simultaneously $\cos(m) > 0$ for $\left(2k - \frac{1}{2}\right)\pi < m < (2k + \frac{1}{2})\pi$ and $\cos(m) < 0$ for $\left(2k + \frac{1}{2}\right)\pi < m < (2k + \frac{3}{2})\pi$.

According to the task $\sin(m) > 0$ so $2\pi k < m < \pi + 2\pi k$, then
$$\pi k < \frac{m}{2} < \frac{\pi}{2} + \pi k \tag{2}$$

When k is even we obtain $\cos(m) > 0$ according to (1), namely
$$\cos(m) = \sqrt{1 - (\sin(m))^2} = \sqrt{1 - 0.11^2} \approx 0.9939 \text{ and } \sin(m/2) > 0,$$
$$\sin\left(\frac{m}{2}\right) = \sqrt{\frac{1 - \cos(m)}{2}} \approx 0.0551.$$

When k is odd, we conclude $\cos(m) < 0$ according to (1), namely
$$\cos(m) = -\sqrt{1 - (\sin(m))^2} = -\sqrt{1 - 0.11^2} \approx -0.9939 \text{ and } \sin(m/2) > 0,$$
$$\sin\left(\frac{m}{2}\right) = \sqrt{\frac{1 - \cos(m)}{2}} \approx 0.9985.$$

Answer: D. 0.05 in case $m = \sin^{-1}(0.11) + 2\pi k$, k is integer.