If $y = e^{ax}\cos^3 x \sin^2 x$, find $\frac{dy}{dx}$

Solution:

We will use next differentiation rules

$$(f(x) * g(x))' = f'(x) * g(x) + f(x) * g'(x) \qquad (1)$$

$$(f(g(x))' = f'(x)g'(x) \qquad (2)$$

So from the first rule we have

$$y' = (e^{ax}\cos^3 x \sin^2 x)' = \left(e^{ax}(\cos^3 x \sin^2 x)\right)' = (e^{ax})'\cos^3 x \sin^2 x + e^{ax}(\cos^3 x \sin^2 x)'$$
$$= (e^{ax})'\cos^3 x \sin^2 x + e^{ax}((\cos^3 x)'\sin^2 x + \cos^3 x(\sin^2 x)')$$

From the chain rule (second rule):

$$(e^{ax})' = e^{ax} * (ax)' = ae^{ax}$$

$$(\cos^3 x)' = 3\cos^2 x * (\cos x)' = -3\cos^2 x * \sin x$$

$$(\sin^2 x)' = 2\sin x * (\sin x)' = 2\sin x * \cos x$$

So:

$$y' = (e^{ax})'\cos^3 x \sin^2 x + e^{ax}((\cos^3 x)'\sin^2 x + \cos^3 x(\sin^2 x)')$$
$$= ae^{ax}\cos^3 x \sin^2 x + e^{ax}(-3\cos^2 x * \sin x * \sin^2 x + \cos^3 x * 2\sin x * \cos x)$$
$$= ae^{ax}\cos^3 x * \sin^2 x - 3e^{ax}\cos^2 x * \sin^3 x + 2e^{ax}\cos^4 x * \sin x$$

Answer: $\frac{dy}{dx} = ae^{ax}\cos^3 x * \sin^2 x - 3e^{ax}\cos^2 x * \sin^3 x + 2e^{ax}\cos^4 x * \sin x$