Prove that in any triangle the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square on the median bisecting the third side

Solution:

Let the triangle have sides *a*, *b*, *c* with a median *d* drawn to side *a*. Let *m* be the length of the segments of *a* formed by the median, so *m* is half of *a*. Let the angles formed between *a* and *d* be θ and θ' where θ includes *b* and θ' includes *c*. Then θ' is the supplement of θ and $\cos\theta' = -\cos\theta$. The law of cosines for θ and θ' states:

$$b^2 = m^2 + d^2 - 2dm\cos\theta \ (1)$$

$$c^2 = m^2 + d^2 - 2dm\cos\theta' = m^2 + d^2 + 2dm\cos\theta \ (2)$$

Add these equations:

$$(1) + (2): b^2 + c^2 = m^2 + d^2 - 2dm\cos\theta + m^2 + d^2 + 2dm\cos\theta =$$

$$= 2m^2 + 2d^2 = 2(m^2 + d^2);$$

$$b^2 + c^2 = 2(m^2 + d^2)$$

Answer: $b^2 + c^2 = 2(m^2 + d^2)$