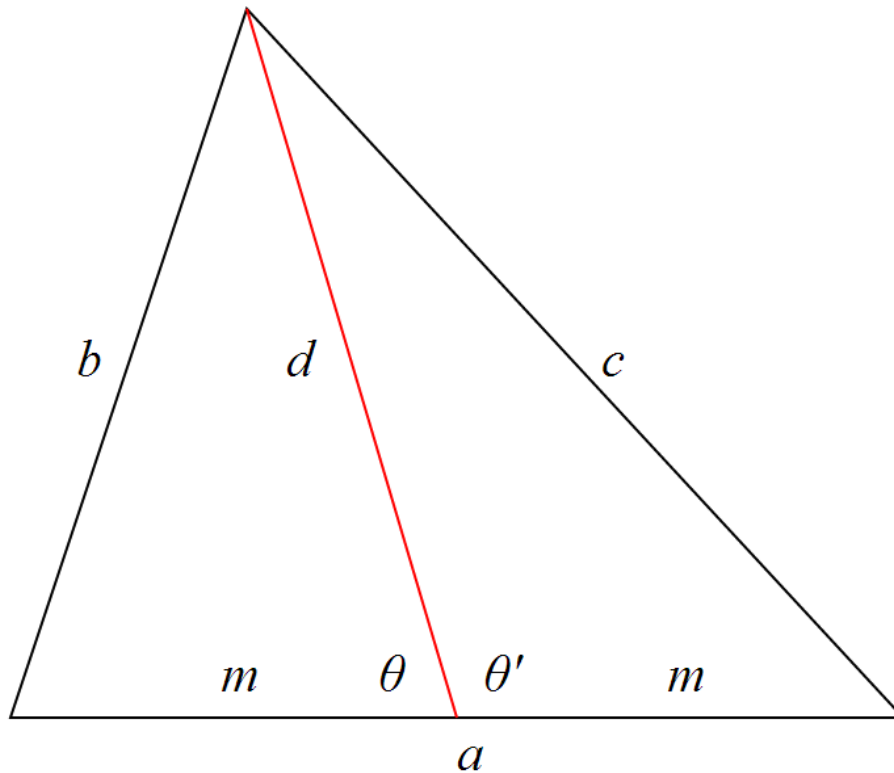**Solution.**

Let the triangle has sides a, b, c with a median d drawn to side a. Let m be the length of the segments of a formed by the median, so m is half of a. Let the angles formed between a and d be θ and θ' where θ includes b and θ' includes c. Then θ' is the supplement of θ and $\cos \theta' = -\cos \theta$. The law of cosines for θ and θ' states

$$b^2 = m^2 + d^2 - 2dm \cos \theta$$

$$c^2 = m^2 + d^2 - 2dm \cos \theta' = m^2 + d^2 + 2dm \cos \theta$$

Add these equations:

$$b^2 + c^2 = 2m^2 + 2d^2$$

So

$$b^2 + c^2 - 2m^2 = 2d^2 \Rightarrow \boldsymbol{b^2 + c^2 \geq 2d^2}$$