

Task. The coordinates of a point A_n are $(2^n, 2^n + 1)$. Find $|A_1A_5|^2$.

Solution. The square of the distance between points $P(x, y)$ and $Q(x_1, y_1)$ is given by the formula:

$$|PQ|^2 = (x - x_1)^2 + (y - y_1)^2.$$

In our case

$$A_1(2^1, 2^1 + 1) = (2, 3), \quad A_5(2^5, 2^5 + 1) = (32, 33),$$

whence

$$|A_1A_5|^2 = (32 - 2)^2 + (33 - 3)^2 = 30^2 + 30^2 = 900 + 900 = 1800.$$

Answer. 1800.