Answer on Question #33354 – Math – Analytic Geometry

Question

Coordinates of the point A_n are $(n^2, 2n)$ and of B_n are $(n^2, -2n)$.

What is the area of the quadrilateral with vertices $A_1B_1A_nB_n$?

Solution

Suppose that n is a natural number.

Coordinates of point A_1 is $(1^2, 2 \cdot 1)$, that is (1,2).

B_1 $(1^2, -2 \cdot 1)$, that is (1,-2).

A_n $(n^2, 2n)$

B_n $(n^2, -2n)$

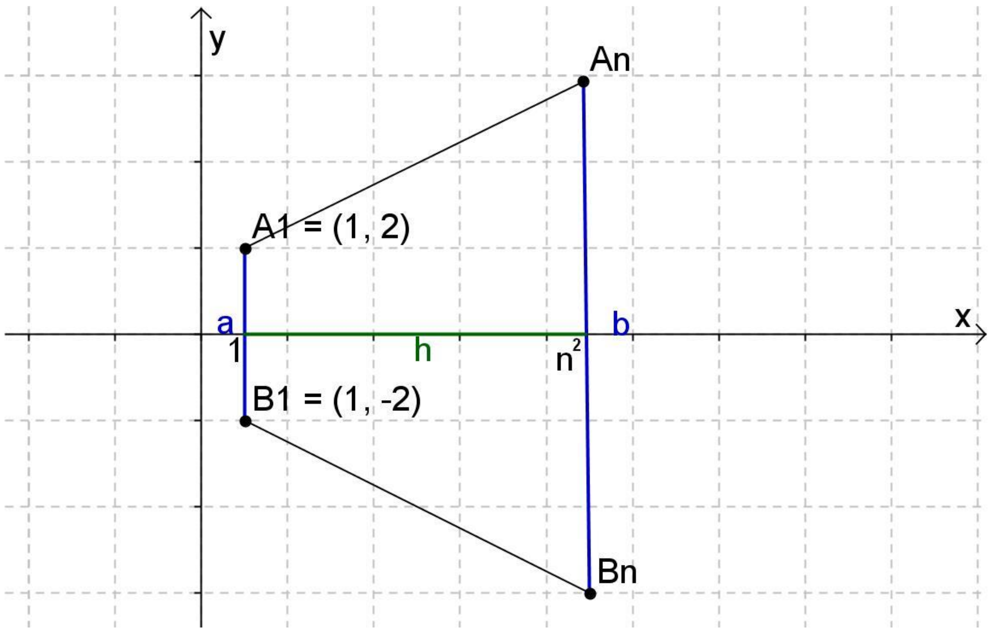
If $n = 1$ than A_1B_1 is a line segment and its area is 0.

If $n > 1$ than $A_1B_1A_nB_n$ is trapezoid.

Area of trapezoid is given by the formula

$$S = \frac{(a+b)}{2} h,$$

where S is area, *a* and *b* are bases of trapezoid, and *h* is height (altitude).



$$h = x_{A_n} - x_{A_1} = n^2 - 1$$

$$a = y_{A_1} - y_{B_1} = 2 - (-2) = 4$$

$$b = y_{A_n} - y_{B_n} = 2n - (-2n) = 4n$$

So,

$$S = \frac{(a+b)}{2}h$$

$$S = \frac{(4+4n)}{2}(n^2 - 1)$$

$$S = 2(n+1)(n^2 - 1)$$

Answer:

$$S = \begin{cases} 2(n+1)(n^2-1), & if\ n > 1, \\ 0, & if\ n = 1. \end{cases}$$