33278:

Task.

a)what is the largest interval (a,b) where the Power series sigma from n=1 to infinity (x-5)^n/(2^(2n)root(n+1)) converges absolutely? [use the Ratio Test]

b) what is the radius of convergence of this Power series sigma from n=1 to infinity (x-5)^n/(2^(2n)root(n+1))?

c) Does the series converge (conditionally) at the endpoints of the interval?

Solution. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^{2n}\sqrt{n+1}}. \tag{3}$$

where $u_n = \frac{(x-5)^n}{2^{2n}\sqrt{n+1}}$

To find the largest interval (a, b) where the power series (3) converges absolutely we use the following ratio test:

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(x-5)^{n+1}2^{2n}\sqrt{n+1}}{2^{2n+2}(x-5)^n\sqrt{n+2}}\right| = \frac{|x-5|}{4} < 1\ . \tag{4}$$

Solving inequality at (4) we get $|x-5| < 4$ which is equivalent to $-4 < x-5 < 4$ and finally $1 < x < 9$. Thus $(a, b) = (1,9)$, the radius of convergence of this power series is $R = 4$.

Check whether the series converges (conditionally) at the endpoints of the interval.

At $x = 1$: the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ is conditionally convergent at this endpoint by Leibniz theorem (all terms $\frac{1}{\sqrt{n+1}}$ decrease, $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$), is not absolutely convergent because the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}} \tag{5}$$

is divergent (apply a generalization of the harmonic series).

At $x = 9$: we come to conclusion that the series is (5), divergent.

The power series (3) converges conditionally at the endpoint $x = 1$.

Answer :

a)interval (1,9);

b)4 ;

c)the series converge (conditionally) at the endpoint x=1 and diverge at the endpoint x=9.