$$\sum_{n=0}^{+\infty}\frac{1}{\sqrt{n^3+2}}=\frac{1}{2}+\sum_{n=1}^{+\infty}\frac{1}{\sqrt{n^3+2}}$$

$\lim\limits_{n\to+\infty}\frac{1}{\sqrt{n^3+2}}=\lim\limits_{n\to+\infty}\frac{1}{\sqrt{n^3}}=\lim\limits_{n\to+\infty}\frac{1}{n^{\frac{3}{2}}}$. This means that the series $\sum_{n=1}^{+\infty}\frac{1}{\sqrt{n^3+2}}$ converges or diverges the same as the series $\sum_{n=1}^{+\infty}\frac{1}{n^{3/2}}$, which converges. So, $\sum_{n=0}^{+\infty}\frac{1}{\sqrt{n^3+2}}=\frac{1}{2}+\sum_{n=1}^{+\infty}\frac{1}{\sqrt{n^3+2}}$ converges.

$$\sum_{n=1}^{+\infty}\frac{3^{n+1}2^{2n}}{4^{2n-1}}$$

$$\lim_{n\to+\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to+\infty}\frac{\frac{3^{n+2}2^{2n+2}}{4^{2n+1}}}{\frac{3^{n+1}2^{2n}}{4^{2n-1}}}=\lim_{n\to+\infty}\frac{3^{n+2}2^{2n+2}4^{2n-1}}{4^{2n+1}3^{n+1}2^{2n}}=\lim_{n\to+\infty}\frac{3\cdot 2^2}{4^2}=\frac{12}{16}<1$$

Here we used the Dalabmer's test. Consequently, the series converges.

$$\sum_{n=2}^{+\infty}\frac{1}{\sqrt{n}+\sin n}$$

$$\frac{1}{\sqrt{n}+\sin n}\geq\frac{1}{\sqrt{n}+1}$$

$\sum_{n=2}^{+\infty}\frac{1}{\sqrt{n}+1}$ - this series diverges, consequently $\sum_{n=2}^{+\infty}\frac{1}{\sqrt{n}+\sin n}$ diverges; follows from the comparison test.