

Use power series to compute $\sin(26.132742)$. Show and explain your work and give an estimate of the error. [26.132742 is a number ... not in degrees, if you feel more comfortable you can read it as 26.132742 radians.]

Definition of the sine:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

26.132742 *radians* = $8\pi + 1$, therefore, $\sin(26.132742) = \sin(8\pi + 1) = \sin(1)$

$$\sin 1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

For example, if $n_{max} = 4$:

$$\begin{aligned} \sin 1 &= \sum_{n=0}^4 \frac{(-1)^n}{(2n+1)!} = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} + O\left(\frac{1}{11!}\right) = \frac{305353}{362880} \\ &= 0.84147101 \end{aligned}$$

An estimate of the error is $O\left(\frac{1}{11!}\right) = \frac{1}{11!} = 2.5 \times 10^{-8}$

So: $\sin(26.132742) = 0.84147101$