

Task:

Solve the following linear programming problem using graphical methods

Minimize $z = 2x + 8y$

Subject to $x - y > -7$

$3x + 2y > 24$

$x > 0$

$y > 0$

Find the minimum z value and name the points

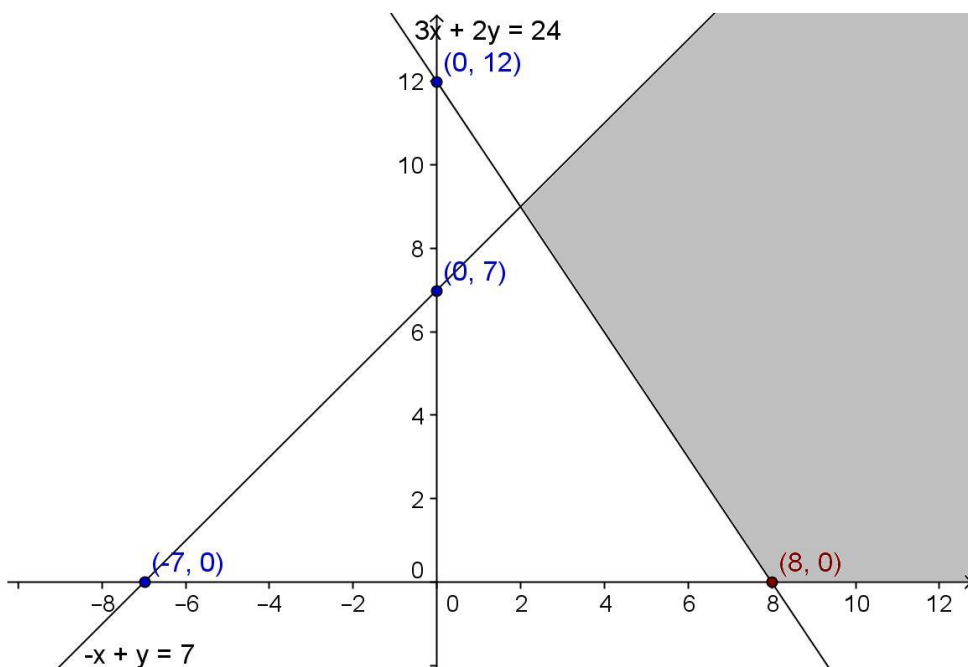
Solution.

1. Depict straight lines $x - y = -7$ and $3x + 2y = 24$

$x - y = -7 \Leftrightarrow y = x + 7$. Put $x = 0$ and $x = -7$. $x - y = -7$ passes through the points $(0, 7)$ and $(-7, 0)$.

$3x + 2y = 24$ passes through the points $(8, 0)$ and $(0, 12)$.

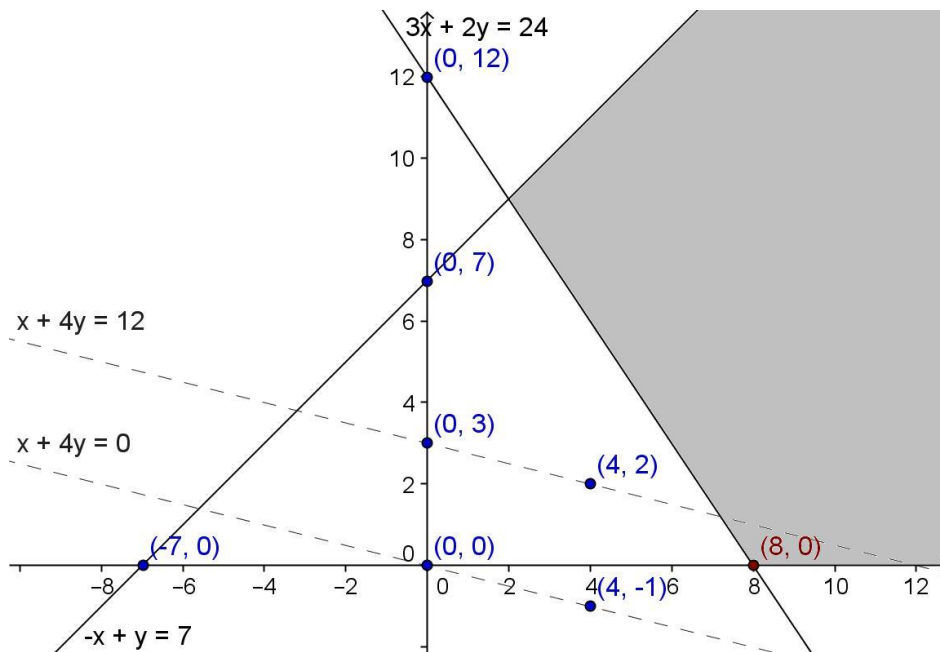
Conditions $x - y > -7$, $3x + 2y > 24$, $x > 0$, $y > 0$ correspond to part of the plane, highlighted in grey.



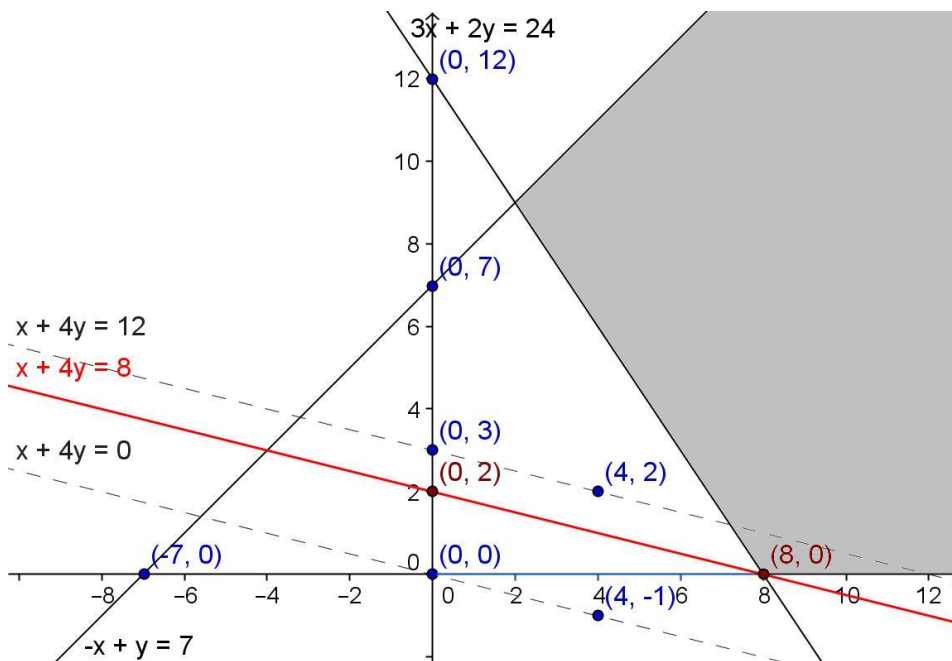
2. $z = 2x + 8y \Leftrightarrow 4y = 1/2 * z - x \Leftrightarrow y = 1/8 * z - 1/4 * x$.

All lines $z = 2x + 8y$ will have the same slope and are moved up or down depending on the value of z .

For example, $z = 0$, $x + 4y = 0$ or $z = 24$, $x + 4y = 12$.



3. As we can see, in order to satisfy the conditions and for minimize $z = 2x + 8y$, this line should cross with the gray part of the plane and pass through the point $(8, 0)$.



Our sought-for line is highlighted in red.

Put in equation $z = 2x + 8y$ $x = 8$ and $y = 0$ we get $z = 16$. Minimum value of z is 16, but it is not achieved, since $x > 0$ and $3x + 2y > 24$. So, $z > 16$.

Answer:

$z > 16$. Minimum value of z is 16, but it is not achieved.