

Task:

Solve the following linear programming problem using graphical methods

$$\text{Minimize } z = 2x + 8y$$

$$\text{Subject to } x - y > -7$$

$$3x + 2y > 24$$

$$x > 0$$

$$y > 0$$

Find the minimum z value and name the points

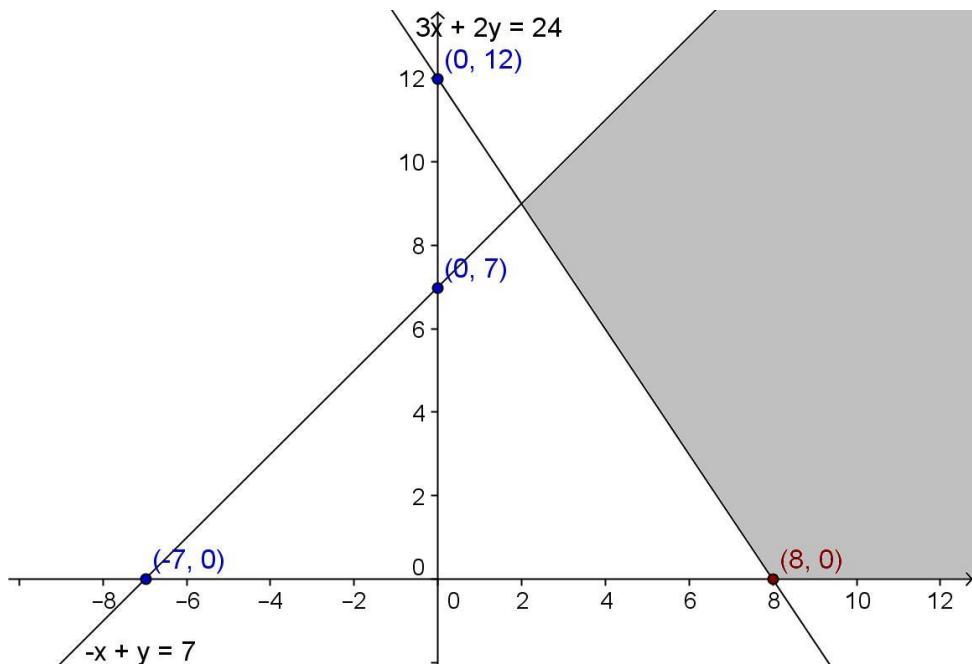
Solution.

1. Depict straight lines $x - y = -7$ and $3x + 2y = 24$

$x - y = -7 \Leftrightarrow y = x + 7$. Put $x = 0$ and $x = -7$. $x - y = -7$ passes through the points $(0, 7)$ and $(-7, 0)$.

$3x + 2y = 24$ passes through the points $(8, 0)$ and $(0, 12)$.

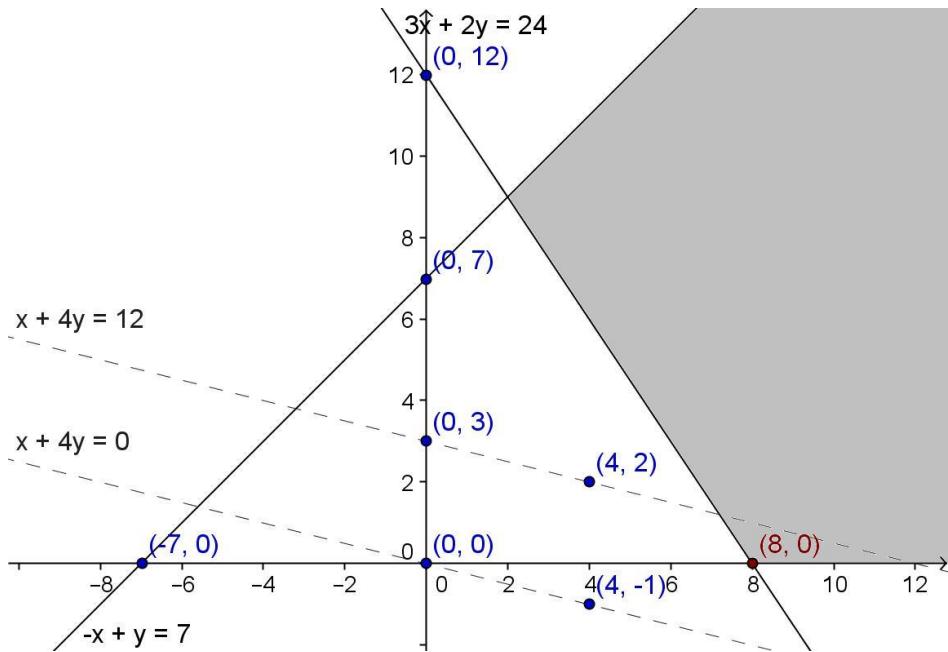
Conditions $x - y > -7$, $3x + 2y > 24$, $x > 0$, $y > 0$ correspond to part of the plane, highlighted in grey.



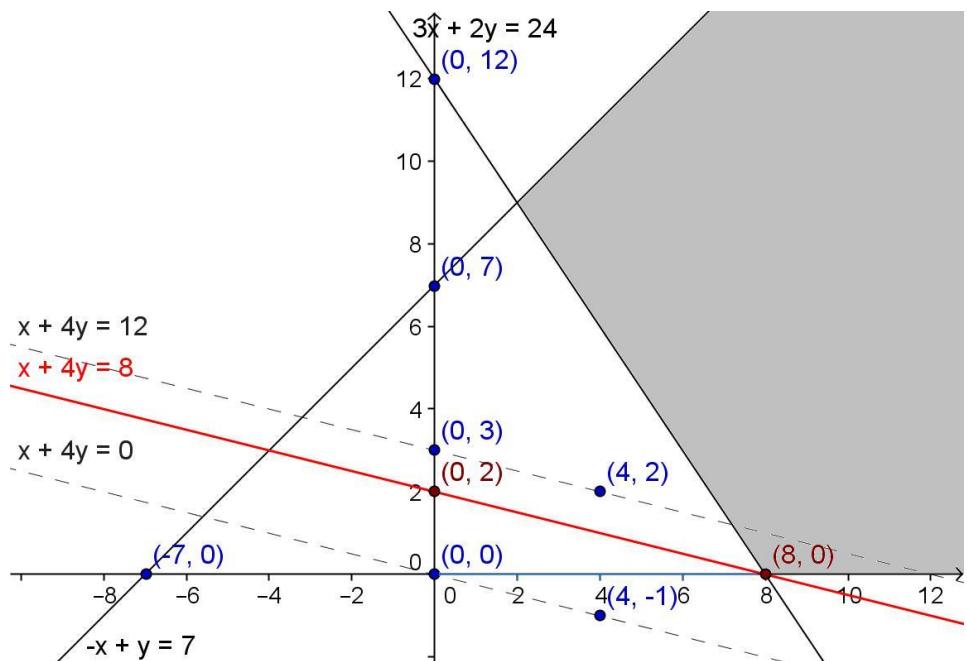
2. $z = 2x + 8y \Leftrightarrow 4y = 1/2z - x \Leftrightarrow y = 1/8z - 1/4x$.

All lines $z = 2x + 8y$ will have the same slope and are moved up or down depending on the value of z .

For example, $z = 0$, $x + 4y = 0$ or $z = 24$, $x + 4y = 12$.



3. As we can see, in order to satisfy the conditions and for minimize $z = 2x+8y$, this line should cross with the gray part of the plane and pass through the point $(8,0)$.



Our sought-for line is highlighted in red.

Put in equation $z=2x+8y$ $x=8$ and $y=0$ we get $z=16$. Minimum value of z is 16, but it is not achieved, since $x>0$ and $3x+2y>24$. So, $z>16$.

Answer:

$z>16$. Minimum value of z is 16, but it is not achieved.