We gather complete squares in left part of equality

$$4y^2+9x^2+16y+18x=k$$
$$4\left(y^2+4y+4\right)-16+9\left(x^2+2x+1\right)-9=k$$
$$4\left(y+2\right)^2+9\left(x+1\right)^2=k+25$$

So, this equation must describe an ellipse. For any $k>-25$ we can perform the following:

$$\frac{4\left(y+2\right)^2}{k+25}+\frac{9\left(x+1\right)^2}{k+25}=1;$$
$$\frac{\left(y+2\right)^2}{\frac{k+25}{4}}+\frac{\left(x+1\right)^2}{\frac{k+25}{9}}=1;$$
$$\frac{\left(y+2\right)^2}{\left(\sqrt{\frac{k+25}{4}}\right)^2}+\frac{\left(x+1\right)^2}{\left(\sqrt{\frac{k+25}{9}}\right)^2}=1;$$

The last equation describes an ellipse with semi-axes $\sqrt{\frac{k+25}{4}},\sqrt{\frac{k+25}{9}}$, because of $k>-25$, they have positive values. Otherwise this values would be zero or even complex, that's why this equation would not be an equation of an ellipse.