Find derivatives of the functions

1) $\cot(\sec x) \cdot \sinh(\log x^2) + \cos x^3$

2) x^x

3) $\exp\left(\tan x^{\sin x}\right)$

Solution.

1. $y(x) = \cot(\sec x) \cdot \sinh(\log x^2) + \cos x^3$

$y'(x) = (\cot(\sec x) \cdot \sinh(\log x^2) + \cos x^3)' = (\cos x^3)' + (\cot(\sec x) \cdot \sinh(\log x^2))' = -\sin x^3 \cdot (x^3)' +$

$+(\cot(\sec x))' \cdot \sinh(\log x^2) + \cot(\sec x) \cdot (\sinh(\log x^2))' \equiv d_1 + d_2 + d_3$

$d_1 = -\sin x^3 \cdot (x^3)' = -3x^2 \sin x^3$

$$d_2 = (\cot(\sec x))' \cdot \sinh(\log x^2) = \frac{1}{\sin^2(\sec x)} \cdot (\sec x)' \cdot \sinh(\log x^2) = -\frac{\sinh(\log x^2)}{\sin^2(\sec x)} \cdot \tan x \sec x =$$

$$= -\csc^2(\sec x) \tan x \sec x \cdot \sinh(\log x^2) = \left|\sinh(\log x^2) = \frac{x^4 - 1}{2x^2}\right| = \csc^2(\sec x) \tan x \sec x \cdot \frac{1 - x^4}{2x^2}$$

$$d_3 = \cot(\sec x) \cdot (\sinh(\log x^2))' = \cot(\sec x) \cdot \cosh(\log x^2) \cdot (\log x^2)' = \cot(\sec x) \cosh(\log x^2) \cdot \frac{1}{x^2} \cdot 2x =$$

$$= \frac{2}{x} \cot(\sec x) \cosh(\log x^2) = \left|\cosh(\log x^2) = \frac{x^2}{2} + \frac{1}{2x^2}\right| = \frac{2}{x} \cot(\sec x) \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) =$$

$$= \cot(\sec x)\left(x + \frac{1}{x^3}\right)$$

So

$$y'(x) = d_1 + d_2 + d_3 = -3x^2 \sin x^3 + \csc^2(\sec x) \tan x \sec x \cdot \frac{1 - x^4}{2x^2} + \cot(\sec x)\left(x + \frac{1}{x^3}\right)$$

2. $y(x) = x^x = \exp(\log x^x) = e^{x \log x}$

$$y'(x) = \left(e^{x \log x}\right)' = e^{x \log x} \cdot (x \log x)' = e^{x \log x} \cdot [x' \log x + x(\log x)'] = e^{x \log x} \cdot [\log x + 1] =$$

$$= x^x [\log x + 1]$$

3. $y(x) = \exp\left(\tan x^{\sin x}\right)$

$$y'(x) = \left(\exp\left(\tan x^{\sin x}\right)\right)' = \exp\left(\tan x^{\sin x}\right) \cdot \left(\tan x^{\sin x}\right)' = \exp\left(\tan x^{\sin x}\right) \cdot \sec^2\left(x^{\sin x}\right) \cdot \left(x^{\sin x}\right)'$$

Let's find $\left(x^{\sin x}\right)'$:

$$\left(x^{\sin x}\right)' = \left(\exp\left(\log x^{\sin x}\right)\right)' = \left(e^{\sin x \log x}\right)' = e^{\sin x \log x} \cdot (\sin x \log x)' = e^{\sin x \log x} \cdot [(\sin x)' \log x +$$

$$+ \sin x \, (\log x)'] = e^{\sin x \log x} \cdot \left[\cos x \log x + \frac{\sin x}{x}\right] = x^{\sin x} \left[\cos x \log x + \frac{\sin x}{x}\right]$$

Then

$$y'(x) = \exp\left(\tan x^{\sin x}\right) \sec^2\left(x^{\sin x}\right) x^{\sin x} \left[\cos x \log x + \frac{\sin x}{x}\right]$$

Answer:

1) $y'(x) = -3x^2 \sin x^3 + \csc^2(\sec x) \tan x \sec x \cdot \frac{1 - x^4}{2x^2} + \cot(\sec x) \left(x + \frac{1}{x^3}\right)$

2) $y'(x) = x^x(\log x + 1)$

3) $y'(x) = \exp\left(\tan x^{\sin x}\right) \sec^2\left(x^{\sin x}\right) x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x}\right)$