

Question 32904

$$\cos x + \sin x + \operatorname{ctg} x = \csc x .$$

This identity is not equal for all x values.

One might solve it, by first rewriting it as $\cos x + \sin x + \frac{\cos x}{\sin x} - \frac{1}{\sin x} = 0$, or

$\cos x + \sin x = \frac{1 - \cos x}{\sin x}$. Multiplying both sides by $\sin x \neq 0$, obtain

$\cos x \sin x + \sin^2 x = 1 - \cos x$, or $\cos x \sin x - \cos^2 x = -\cos x$, from which $\sin x - \cos x = -1$.
Formally, this equation has two roots $x=0$ and $x=\frac{-\pi}{2}$.

But substituting it into $\cos x + \sin x + \operatorname{ctg} x = \csc x$, check that only $x = -\frac{\pi}{2}$ is the solution.