If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

Solution:

Express the function *y* as a function of one variable *y(x):*

$$x\sqrt{1+y} = -y\sqrt{1+x} \ (1)$$

Bring both sides of the square (and then choose the appropriate roots):

$$x^2(1+y) = y^2(1+x)$$

$$y^2(1+x) - x^2y - x^2 = 0$$

Got a quadratic equation (variable y), find the roots of a quadratic equation using the formula:

$$y = \frac{x^2 \pm \sqrt{x^4 + 4x^2(1+x)}}{2(1+x)}$$

$$y_1 = \frac{2x^2 + 2x}{2x+2} = x$$

$$y_2 = \frac{x^2 - x^2 - 2x}{2x+2} = -\frac{x}{x+1}$$

The first root is wrong, because when we brought both sides of equation to square, we have ignored the sign (minus) (1):

$$y = y_2 = -\frac{x}{x+1}$$

$$\frac{dy}{dx} = y' = -\left(\frac{x}{x+1}\right)'$$

Apply the quotient rule:

$$y' = -\left(\frac{x}{x+1}\right)' = -\frac{x'(x+1) - (x+1)'x}{(x+1)^2} = -\frac{x+1-x}{(x+1)^2} = -\frac{1}{(x+1)^2}$$

$$y' = \frac{dy}{dx} = -\frac{1}{(1+x)^2}$$

Answer: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$