

If  $\sec\theta - \tan\theta = p$  find value of  $\sec\theta$  and  $\tan\theta$ .

**Solution:**

Using a well-known trigonometric equation:

$$1 + \tan^2\theta = \sec^2\theta$$

we will have that

$$\sec^2\theta - \tan^2\theta = 1$$

$$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$

from of the problem we know that

$$\sec\theta - \tan\theta = p$$

so

$$p * (\sec\theta + \tan\theta) = 1$$

where we can find that

$$\sec\theta + \tan\theta = \frac{1}{p}$$

So we have system of two equations

$$\begin{cases} \sec\theta - \tan\theta = p \\ \sec\theta + \tan\theta = \frac{1}{p} \end{cases}$$

If we sum them we will find  $\sec\theta$ , if we subtract from one second we will find  $\tan\theta$

$$\begin{cases} 2\sec\theta = p + \frac{1}{p} \\ -2\tan\theta = p - \frac{1}{p} \end{cases}$$

So

$$\begin{cases} \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) \\ \tan\theta = \frac{1}{2}\left(\frac{1}{p} - p\right) \end{cases}$$

**Answer:**  $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$        $\tan\theta = \frac{1}{2}\left(\frac{1}{p} - p\right)$