

Task. The point $P(x, 3)$ is equidistant from $A = (-4, 7)$ and $B = (5, -6)$. Find x ?

Solution. We have that $AP = BP$. Let us write squares of distances:

$$AP^2 = (-4 - x)^2 + (7 - 3)^2 = (4 + x)^2 + 4^2 = 16 + 8x + x^2 + 16 = x^2 + 8x + 32$$

$$BP^2 = (5 - x)^2 + (-6 - 3)^2 = (5 - x)^2 + 9^2 = 25 - 10x + x^2 + 81 = x^2 - 10x + 106$$

Then from $AP^2 = BP^2$ we obtain

$$x^2 + 8x + 32 = x^2 - 10x + 106$$

$$8x + 32 = -10x + 106$$

$$8x + 10x = 106 - 32$$

$$18x = 106 - 32 = 74$$

$$x = \frac{74}{18} = \frac{37}{9}.$$

Answer. $x = \frac{37}{9}$.