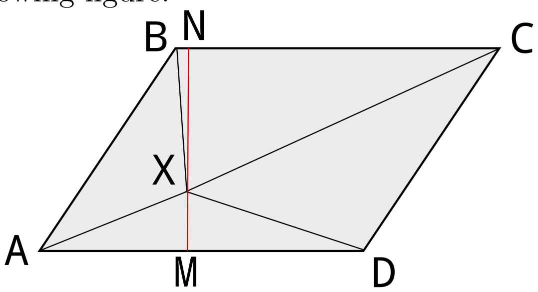
Task. Given parallelogram ABCD, choose any point in the interior of the parallelogram and call it X. Connect point A,B,C,D to X. Show the sum of areas of diagonal nonadjacent triangles are equal.

Proof. Consider the following figure:



We should prove the sums of areas of opposite triangles are equal

$$S_{ABX} + S_{CDX} = S_{ADX} + S_{BCX}.$$

Since the sum of all triangles is equal to the area of parallelogram $ABCD$:

$$S_{ABX} + S_{CDX} + S_{ADX} + S_{BCX} = S_{ABCD}$$

it suffices to prove that one of sums is equal to the half of S_{ABCD}:

$$S_{ADX} + S_{BCX} = \frac{1}{2}\ S_{ABCD}.$$

Recall that the area of triangle with the side a and the height to that side is equal to

$$S = \frac{1}{2}\ ah.$$

Let XM be the height in the triangle ADX and XN be the height in the triangle BCX. Then

$$S_{ADX} = \frac{1}{2}AD * XM, \qquad S_{BCX} = \frac{1}{2}BC * XN.$$

Since $ABCD$ is a parallelogram, $AD = BC$. Moreover, the sides AD and BC are parallel, so MN is perpendicular to AD and BC, and so MN is the height in the parallelogram $ABCD$. In particular, $X \in MN$, and thus $MN = XN + XM$. Therefore

$$S_{ADX} + S_{BCX} = \frac{1}{2}AD * XM + \frac{1}{2}BC * XN = \frac{1}{2}AD * XM + \frac{1}{2}AD * XN =$$
$$= \frac{1}{2}AD(XM + XN) = \frac{1}{2}AD * MN.$$

On the other hand, it is known that the area of the parallelogram is

$$S_{ABCD} = AD * MN,$$

whence

$$S_{ADX} + S_{BCX} = \frac{1}{2}AD * MN = \frac{1}{2}S_{ABCD}.$$

Therefore

$$S_{ABX} + S_{CDX} = S_{ABCD} - (S_{ADX} + S_{BCX}) = S_{ABCD} - \frac{1}{2}S_{ABCD} = \frac{1}{2}S_{ABCD} = S_{ADX} + S_{BCX}.$$