$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{mn^2+nm^2+2mn}$$

Solution.

Transform the given expression:

$$\frac{1}{mn^2+nm^2+2mn}=\frac{1}{m(n^2+mn+2n)}=\frac{1}{m(m+2)}\frac{m+2}{n^2+mn+2n}$$

Then factorize the obtained expression:

$$\frac{1}{m(m+2)}\frac{m+2+n-n}{n^2+mn+2n}=\frac{1}{m(m+2)}\left(\frac{1}{n}-\frac{1}{m+n+2}\right)$$

We have the following expression:

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{1}{m(m+2)}\right)\left(\frac{1}{n}-\frac{1}{m+n+2}\right)$$

Transform the expression:

$$\sum_{m=1}^{\infty}\left[\left(\frac{1}{m(m+2)}\right)\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{m+n+2}\right)\right]$$

Find this sum:

$$\sum_{m=1}^{\infty}\left[\left(\frac{1}{m(m+2)}\right)\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{m+n+2}\right)\right]=\frac{7}{4}$$

Answer: $7/4$