

Task. A discrete random variable X has possible values $x_i = i^2$, $i = 1, 2, 3, 4, 5$, which occur with probabilities 0.4, 0.25, 0.15, 0.1, and 0.1 respectively.

a) write the probability density $f_X(t)$ and probability distribution $F_X(t)$ functions for X random variable

b) find mean value $E[X]$

Solution. The distribution of probabilities of values of X is given in the following table:

x_i	1	4	9	16	25
$p(x_i)$	0.4	0.25	0.15	0.1	0.1

Since X is discrete, its probability density function is the sum of delta-functions:

$$f_X(t) = 0.4\delta(t - 1) + 0.25\delta(t - 4) + 0.15\delta(t - 9) + 0.1\delta(t - 16) + 0.1\delta(t - 25),$$

and its probability distribution $F_X(t)$ is given by the formula:

$$F_X(t) = \begin{cases} 0, & t < 1 \\ 0.4, & 1 \leq t < 4 \\ 0.4 + 0.25 = 0.65, & 4 \leq t < 9 \\ 0.65 + 0.15 = 0.8, & 9 \leq t < 16 \\ 0.8 + 0.1 = 0.9, & 16 \leq t < 25 \\ 1, & t \leq 25 \end{cases}$$

Thus

$$F_X(t) = \begin{cases} 0, & t < 1 \\ 0.4, & 1 \leq t < 4 \\ 0.65, & 4 \leq t < 9 \\ 0.8, & 9 \leq t < 16 \\ 0.9, & 16 \leq t < 25 \\ 1, & t \leq 25 \end{cases}$$

The mean value $E[X]$ is defined by the formula:

$$\begin{aligned} E[X] &= \sum_{i=1}^5 x_i p(x_i) = 1 * 0.4 + 4 * 0.25 + 9 * 0.15 + 16 * 0.1 + 25 * 0.1 \\ &= 0.4 + 1 + 1.35 + 1.6 + 2.5 = 6.85. \end{aligned}$$