Solve a system of equations:

$$\begin{cases} 3(2u+v)=7uv \ \ (1) \\ 3(u+3v)=11uv \ (2) \end{cases}.$$

Solution.

Assume that $uv=0$. Then we have the following system of equations:

$$\begin{cases} 3(2u+v)=0 \ \ (1) \\ 3(u+3v)=0 \ \ (2) \end{cases} \Leftrightarrow \begin{cases} 2u+v=0 \ \ (1) \\ u+3v=0 \ \ (2) \end{cases} \Leftrightarrow \begin{cases} v=-2u \ \ (1) \\ -5u=0 \ \ (2) \end{cases} \Leftrightarrow \begin{cases} u=0 \ \ (1) \\ v=0 \ \ (2) \end{cases}.$$

If $u=0$ and $v=0$ then $uv=0$, so (0, 0) is a solution of the system.

Assume that $uv\neq 0$, then $u\neq 0$ and $v\neq 0$ and we can divide the first equation of the original system by the second equation:

$$\begin{cases} 3(2u+v)=7uv \ \ (1) \\ 3(u+3v)=11uv \ (2) \end{cases} \Leftrightarrow \begin{cases} \dfrac{2u+v}{u+3v}=\dfrac{7}{11} \qquad (1) \\ 3(u+3v)=11uv \ (2) \end{cases} \Leftrightarrow \begin{cases} 22u+11v=7u+21v \ (1) \\ 3(u+3v)=11uv \qquad (2) \end{cases} \Leftrightarrow \begin{cases} 15u=10v \qquad (1) \\ 3(u+3v)=11uv \ (2) \end{cases} \Leftrightarrow$$

$$\begin{cases} u=\dfrac{2}{3}v \qquad (1) \\ 3\left(\dfrac{2}{3}v+3v\right)=11\cdot\dfrac{2}{3}v^2 \ (2) \end{cases} \Leftrightarrow \begin{cases} u=\dfrac{2}{3}v \qquad (1) \\ 11v=11\cdot\dfrac{2}{3}v^2 \ (2) \end{cases} \Leftrightarrow \begin{cases} u=1 \qquad (1) \\ v=\dfrac{3}{2} \qquad (2) \end{cases}.$$

If $u=1$ and $v=\dfrac{3}{2}$ then $uv\neq 0$, so $\left(1, \dfrac{3}{2}\right)$ is the second solution of the original system.

Answer: $(0,0)$; $\left(1, \dfrac{3}{2}\right)$.