Task. Three children, each accompanied by a guardian, seek admission in a school. The principal wants to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this be done?

Solution. There is no general formula for the solution of this problem, so we will split the computation into a series of special cases.

Let A, B, C be the guardians, and a, b, c be the corresponding childrens. We should order these 6 letters so that A stand before a, B stand before b, and C stand before c. It will be convenient to write $X < Y$ if letter X preceedes letter Y. Thus we should the number N of all orderings in which

$$A < a, \qquad B < b, \qquad C < c.$$

Notice that neither of letters a, b, c can stand at first position, since otherwise it will preceede the corresponding capital letter. Similarly, neither of letters A, B, C can stand at 6-th position, since otherwise it will follow the corresponding small letter.

Let N_{ABC} be the number of orderings in which $A < B < C$. Since the number of orderings of three letters A, B, C is equal to $3! = 6$, we have that

$$N = 6N_{ABC}.$$

Thus we obtain the following cases of positinos of letters A, B, C.

1) $ABC * **$.

Thus A is the first letter, B is the second letter, C is the third letter, and the last three positions can be filled by letters a, b, c in any order. The number of orderings of letters a, b, c is equal to $3! = 6$, so the number of variants in this case is 6.

2) $AB * C * *$.

In this case the letter c can not stand at third position, since otherwise we will have that $c < C$. All other orderings are allowed and we obtain the following 4 variants:

$$ABaCbc, \qquad ABaCcc, \qquad ABbCac, \qquad ABbCca.$$

3) $AB * *C*$.

In this case the letter c must stand at last position, while a and b can stand on 3rd and 4th positions in any order:

$$ABabCc, \qquad ABbaCc.$$

Thus in this case we have 2 variants.

4) $A * BC * *$.

In this case neither of letters b or c can stand at 2nd position, so we obtain the following 2 variants:

$$AaBCbc, \qquad AaBCcb.$$

5) $A * B * C*$.

In this case c must stand at position 6. Therefore b must be at position 4 and therefore a will be at position 2. So we get a unique variants:

$$AaBbCc.$$

6) $A * *BC*$.

This situation is impossible since b and c must stand after B and C, i.e. at position 6.

Thus

$$N_{ABC} = 6 + 4 + 2 + 2 + 1 + 0 = 15.$$

Therefore

$$N = 6 * 15 = 90.$$

Answer. 90.