This is an extremum problem with constrains $a, b, c \geq 0,\ a+b+c-\frac{9}{2}=0$. Consider corresponding Lagrange function

$$L = \frac{a}{b^3+54} + \frac{b}{c^3+54} + \frac{c}{a^3+54} + t(a+b+c-9/2)\ ,$$

here t is Lagrange multiplier. Find partial derivatives of L with respect a, b, c and t them stationary point of L . We have

$$\frac{\partial L}{\partial a} = \frac{1}{b^3+54} - \frac{3a^2c}{(a^3+54)^2} + t = 0$$

$$\frac{\partial L}{\partial b} = \frac{1}{c^3+54} - \frac{3b^2a}{(b^3+54)^2} + t = 0$$

$$\frac{\partial L}{\partial \mathrm{c}} = \frac{1}{a^3+54} - \frac{3c^2b}{(c^3+54)^2} + t = 0$$

$$\frac{\partial L}{\partial t} = a+b+c-9/2 = 0.$$

Solution of this system is $a = b = c = \frac{3}{2}$.

Since $f\left(\frac{3}{2},\frac{3}{2},\frac{3}{2}\right) = \frac{4}{51} > f\left(\frac{3}{2},3,0\right) = 2/27$,

point $M\left(\frac{3}{2},\frac{3}{2},\frac{3}{2}\right)$ is not a minimum point. Put

$$D = \left\{(x,y,z) \in R^3 : x \geq 0, y \geq 0, z \geq 0, x+y+z = \frac{9}{2}\right\}.$$

The boundary of closed set D is a triangle $ABC, A\left(\frac{9}{2},0,0\right), B\left(0,\frac{9}{2},0\right), C\left(0,0,\frac{9}{2}\right)$. Every minimum point of f inside D is a stationary point of Lagrange function. Therefore function f can have a minimum point on the boundary of D only. This boundary consists of segments *AB*, *BC*, *CA*. Let us investigate behavior of f on *AB* since two other cases for *BC* and *CB* can be considered in the same way. Now our problem is equivalent to following problem: minimize the function

$$g(a,b) \coloneqq f(a,b,0) = \frac{a}{b^3+54} + \frac{b}{54},$$

under conditions

$$b \in \left[0,\frac{9}{2}\right], a = \frac{9}{2} - b.$$

Let us substitute expression for *a* into $g(a,b)$. We get

$$F(b) \coloneqq g\left(\frac{9}{2} - b, b\right) = \frac{1}{54}\frac{243 + b^4}{b^3 + 54}.$$

Furthermore

$$\frac{dF}{db} = \frac{1}{54}\frac{b^2(216b - 729 + b^4)}{(b^3 + 54)^2}.$$

There is exactly one root of derivative $\frac{dF}{db}$ in open interval$(0, \frac{9}{2})$. This root is $b = 3$. Because $F'(2) = -\frac{281}{51894} < 0$, and $F'(4) = \frac{782}{93987} > 0$, point $b = 3$ is a local minimum point of F. We have

$$F(3) = f\left(\frac{3}{2}, 3, 0\right) = \frac{2}{27}.$$

For endpoints of interval we obtain

$$F(0) = f\left(\frac{9}{2}, 0, 0\right) = \frac{1}{12} > \frac{2}{27}\ ,$$

$$F\left(\frac{9}{2}\right) = f\left(0, \frac{9}{2}, 0\right) = \frac{1}{12}.$$

Answer: the minimum value of expression $\frac{a}{b^3+54} + \frac{b}{c^3+54} + \frac{c}{a^3+54}$ over all real nonnegative numbers a, b, c such that $a + b + c - \frac{9}{2} = 0$ is equal to $\frac{1}{12}$.