Evaluate 3(x^2) - 4x + 2 dx from first principles (upper bound: 4, lower bound: 2)

Solution.

We have the integral:

$$\int_2^4 (3x^2 - 4x + 2)dx$$

Break it into three simple integrals:

$$\int_2^4 3x^2 dx - \int_2^4 4x dx + \int_2^4 2dx \qquad (1)$$

We know this formula:

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

At first find the value of the first integral using this formula:

$$\int_2^4 3x^2 dx = 3\int_2^4 x^2 dx = 3 \cdot \frac{x^3}{3}\Big|_2^4 = 4^3 - 2^3 = 64 - 8 = 56$$

Then second integral:

$$\int_2^4 4x dx = 4\int_2^4 x dx = 4 \cdot \frac{x^2}{2}\Big|_2^4 = 2 \cdot (4^2 - 2^2) = 24$$

Third integral:

$$\int_2^4 2dx = 2x|_2^4 = 4$$

Finally find the value of the (1) integral:

$$\int_2^4 3x^2 dx - \int_2^4 4x dx + \int_2^4 2dx = 56 - 24 + 4 = 36$$

Answer: 36.