

$$2(ax-by)+(a+4b)=0,$$

$$2(bx+ay)+(b-4a)=0$$

We can rewrite as

$$2a*x - 2b*y = -a-4b$$

$$2b*x+2a*y = 4a - b$$

if $a=b=0$ then we get $0=0$, $0=0$. so $\forall x,y$ (x,y) - solution.

Suppose that $a=0$, $b \neq 0$. Then

$$-2b*y=-4b, 2bx=-b$$

$$y=2, x=-0.5$$

In case $b=0$, $a \neq 0$:

$$2a*x=-a, 2a*y=4a$$

$$x=-0.5, y=2$$

Now we can consider case $a \neq 0$ and $b \neq 0$.

Then from first equation

$$2a*x = 2b*y - a - 4b$$

$$x = \frac{2b*y - a - 4b}{2a}$$

we can substitute x to the second equation

$$2b * \frac{2b*y - a - 4b}{2a} + 2a*y = 4a - b$$

$$\frac{b * (2b*y - a - 4b)}{a} + 2a*y = 4a - b$$

$$\frac{b * 2b}{a} * y - \frac{(a + 4b)b}{a} + 2a*y = 4a - b$$

$$(\frac{b * 2b}{a} + 2a) * y = \frac{(a + 4b)b}{a} + 4a - b$$

$$(\frac{2(a^2 + b^2)}{a}) * y = \frac{(a + 4b)b}{a} + 4a - b$$

$$a^2 + b^2 > 0$$

Then we can divide

$$\begin{aligned}
y &= \left(\frac{(a+4b)b}{a} + 4a - b \right) * \frac{a}{2(a^2+b^2)} = \frac{(a+4b)b + 4a^2 - ab}{a} * \frac{a}{2(a^2+b^2)} = \\
&= \frac{4(a^2+b^2)}{a} * \frac{a}{2(a^2+b^2)} = 2
\end{aligned}$$

$$\text{But } x = \frac{2b*y-a-4b}{2a} = \frac{2b*2-a-4b}{2a} = \frac{4b-a-4b}{2a} = -\frac{a}{2a} = -0.5$$

So in this case $x=-0.5$, $y=2$.

And finally we get that if $a=b=0 \forall x,y$ (x,y) - solution. In other cases $x=-0.5$, $y=2$.