

Task. Three consecutive vertices of a parallelogram are points $A = (2, 4)$, $B = (0, 0)$, and $C = (6, 0)$. Find the fourth vertex D .

Solution. Recall that the intersection point M of the diagonals AC and BD of the parallelogram is the middle point of these intervals.

Thus if $A = (x_1, y_1)$ and $C = (x_2, y_2)$, then the coordinates of the middle point $M = (\bar{x}, \bar{y})$ of AC can be computed by the formula:

$$\bar{x} = \frac{x_1 + x_2}{2}, \quad \bar{y} = \frac{y_1 + y_2}{2}.$$

In our case $A = (2, 4)$ and $C = (6, 0)$, whence

$$\bar{x} = \frac{2 + 6}{2} = 4, \quad \bar{y} = \frac{4 + 0}{2} = 2,$$

so

$$M = (4, 2).$$

Let $D = (x, y)$. Since $M = (4, 2)$ is the middle point of BD and $B = (0, 0)$ we have that

$$4 = \frac{0 + x}{2}, \quad 2 = \frac{0 + y}{2},$$

whence

$$x = 2 * 4 = 8, \quad y = 2 * 2 = 4.$$

Thus $D = (8, 4)$.

Answer. $D = (8, 4)$.