

Prove:

$$\frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$$

Proof:

The Multiplication Properties of Equality:

If you multiply two equality elements by the same element, then the resulting elements are equivalent.

Multiplying both sides by $\frac{\sin A}{1 - \cos A}$

$$\text{LHS: } \frac{\sin A}{1 + \cos A} \cdot \frac{\sin A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos^2 A}$$

The Pythagorean Identities:

$$\sin^2 A + \cos^2 A = 1$$

So $1 - \cos^2 A = \sin^2 A$ and so

$$\text{LHS: } \frac{\sin^2 A}{1 - \cos^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$$

$$\text{RHS: } \frac{1 - \cos A}{\sin A} \cdot \frac{\sin A}{1 - \cos A} = 1$$

Hence

LHS=RHS