

Conditions

Here's the problem

I have a scalene Triangle with Vertices A,B and C. I know the coordinates of A and B. and i know angle CAB and angle CBA. now I have to find the coordinates of C.
please help me with the Solution

Solution

We must use vector interpretation of this problem.

Consider points A and B with known coordinates:

$$A(a_1, a_2), B(b_1, b_2)$$

And point C with unknown coordinates:

$$C(c_1, c_2),$$

Consider vectors:

$$\overrightarrow{AC} = (c_1 - a_1, c_2 - a_2)$$

$$\overrightarrow{BC} = (c_1 - b_1, c_2 - b_2)$$

$$\overrightarrow{AB} = (b_1 - a_1, b_2 - a_2)$$

$$\overrightarrow{BA} = (a_1 - b_1, a_2 - b_2)$$

We know, that the scalar product of two vectors is related to a cosine of an angel between these vectors in the following way:

$$\overrightarrow{AC} \cdot \overrightarrow{AB} = |\overrightarrow{AC}| \cdot |\overrightarrow{AB}| \cdot \cos CAB$$

And

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{AB}| \cdot |\overrightarrow{BC}| \cdot \cos CBA$$

You know the values of both angles; hence, you know their cosines. There are two unknown variables in these two equations - c_1, c_2 . Put these equations to a system, solve it and you will get the coordinates of point C.