Question #31383

a circle has equation (x-2)^2 + (y+3)^"=25
a. find the centre and radius of the cirle
b.verify that the point A(6,-6) is on the circle
c. if [AB] is a diameter of the circle, find point B.
d.Find the equation of the tangent to the circle A

Solution.

a. The circle with a center (x_0, y_0) and radius R has an equation $(x - x_0)^2 + (y - y_0)^2 = R^2$.
Then the center of the given circle is $O(2, -3)$ and the radius R=$\sqrt{25} = 5$.

Answer. (2,-3), R=5.

b. A point P(x, y) is on the circle if its coordinates satisfy the equation of this circle. Then $(6 - 2)^2 + (-6 + 3)^2 = 4^2 + (-3)^2 = 16 + 9 = 25$, which means that A(6,-6) is on the circle.

c. Let $B(x_0, y_0)$. Then B is on the circle, which means that $(x_0 - 2)^2 + (y_0 + 3)^2 = 25$ and $AB = \sqrt{(6 - x_0)^2 + (-6 - y_0)^2} = 2R = 10$. Thus, we obtain

$$\begin{cases} (x_0 - 2)^2 + (y_0 + 3)^2 = 25 \\ (6 - x_0)^2 + (-6 - y_0)^2 = 100 \end{cases}$$

Then

$$\begin{cases} x_0^2 - 4x_0 + 4 + y_0^2 + 6y_0 + 9 = 25 \\ 36 - 12x_0 + x_0^2 + 36 + 12y_0 + y_0^2 = 100 \end{cases}$$

Subtracting the equalities, we obtain $x_0 = \frac{3y_0 - 8}{4}$ and
$25y_0^2 = 0$. Then $y_0 = 0, x_0 = -2$.

Answer. (-2,0).

d. A tangent line to the circle at a point A is perpendicular to the line, which contains the center of the circle and A.
The equation of the line through the points A and O:

$$\frac{x - 6}{4} = \frac{y + 6}{-3}$$

and so

$$3x + 4y + 6 = 0$$

It follows that a tangent line is parallel to the vector (3,4) and so $\frac{x-6}{3} = \frac{y+6}{4}$
The tangent line $4x - 3y - 42 = 0$.

Answer. $4x - 3y - 42 = 0$.