

Task. Antiderivative of $f(x) = (2x + 1)^{1/2}$.

Solution. The antiderivative of the function $f(x) = (2x + 1)^{1/2}$ is the indefinite integral

$$F(x) = \int (2x + 1)^{1/2} dx$$

Let us change variable: $t = 2x + 1$. Then $dt = (2x + 1)'dt = 2dx$. Hence $dx = \frac{dt}{2}$ and therefore

$$\begin{aligned} F(x) &= \int (2x + 1)^{1/2} dx = \int t^{1/2} \frac{dt}{2} = \frac{1}{2} \int t^{1/2} dt = \frac{1}{2} * \frac{t^{1/2+1}}{1/2+1} + C \\ &= \frac{1}{2} * \frac{t^{3/2}}{3/2} + C = \frac{t^{3/2}}{3} + C. \end{aligned}$$