Task. If f is a differentiable function such that $f(3) = 8$ and $f'(3) = 5$, which of the following statements could be false?

a) $\lim\limits_{x\to 3} f(x) = 8$

b) $\lim\limits_{x\to 3} \frac{f(x)-8}{x-3} = 5$

c) $\lim\limits_{h\to 0} \frac{f(3+h)-8}{h} = 5$

d) $\lim\limits_{x\to 3+} f(x) = \lim_{x\to 3-} f(x)$

e) $\lim\limits_{x\to 3} f'(x) = 5$

Solution. Notice that f is differentiable at $x = 3$, it is also continuous at this point. This means that

$$\lim_{x\to 3} f(x) = \lim_{x\to 3+} f(x) = \lim_{x\to 3-} f(x) = f(3) = 8,$$

whence a) and d) hold.

Moreover, differentiability of f at $x = 3$ means that

$$f'(3) = \lim_{x\to 3} \frac{f(x) - 8}{x - 3} = 5,$$

whence b) also holds.

In particular, the later limit hold if we approach x from the left, i.e.

$$\lim_{h\to 0} \frac{f(3+h) - 8}{h} = 5$$

and thus c) holds.

On the other hand, statement e)

$$\lim_{x\to 3} f'(x) = f'(3) = 5$$

means that f' is continuous at $x = 3$. However, the differentiability of f at $x = 3$ does not imply that f' is not continuous at $x = 3$. Therefore statement e) could be false.