Find the angle between U =4i - 2j + 4k and V = 3i - 6j - 2k.

Solution

In our case we have such vectors: $\vec{U}(4;-2;4)\ and\ \vec{V}(3;-6;-2)$.

To find the angle between them we must find the cosine of that angle at first.

This cosine is equal to scalar multiplication of those vectors, divided by multiplication of sizes of those vectors.

The scalar multiplication of two vectors is equal to sum of multiplications of corresponding coordinates. $\vec{U}\cdot\vec{V}=u_1v_1+u_2v_2+u_3v_3$, where u_1,u_2,u_3 are the coordinates of the first vector and v_1,v_2,v_3 are the coordinates of the second one.

The size of a vector is equal to square root from sum of squares of its coordinates.

$$|\vec{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$$

$$\cos\left(\widehat{\vec{U};\vec{V}}\right)=\frac{\vec{U}\cdot\vec{V}}{|\vec{U}|\cdot|\vec{V}|}=\frac{u_1v_1+u_2v_2+u_3v_3}{\sqrt{u_1^2+u_2^2+u_3^2}\cdot\sqrt{v_1^2+v_2^2+v_3^2}}$$

$$\cos\left(\widehat{\vec{U};\vec{V}}\right)=\frac{4\cdot3+2\cdot6-4\cdot2}{\sqrt{16+4+16}\cdot\sqrt{9+36+4}}=\frac{16}{42}=\frac{8}{21}.$$

Now when we have the cosine we can find the corresponding angle (using calculator or tables)

$$\arccos\left(\frac{8}{21}\right)\approx\arccos(0.38095)\approx67.6°=67°36'.$$

$67°36'$ is closer to $68°$ than to $67°$.