Integrate with respect to x: ∫ x^2/((x-2)(x^2+1)) dx
4/5 ln(x-2)+2/5 tan^(-1) x+c
4/5 ln(x-2)+1/5 ln(x^2+1)+2/5 tan^(-1) x+c

Solution

$$\boldsymbol{I} = \int \frac{\mathrm{x}^2}{(\mathrm{x}-2)(\mathrm{x}^2+1)}\,\mathrm{dx} = \int \frac{\mathrm{x}^2+1-1}{(\mathrm{x}-2)(\mathrm{x}^2+1)}\,\mathrm{dx} = \int \left(\frac{1}{(\mathrm{x}-2)} - \frac{1}{(\mathrm{x}-2)(\mathrm{x}^2+1)}\right)dx$$

$$\frac{1}{(\mathrm{x}-2)(\mathrm{x}^2+1)} = \frac{A}{(\mathrm{x}-2)} + \frac{Bx+C}{(\mathrm{x}^2+1)}$$

Let’s find A,B,C:

$$x^2: A+B=0 \rightarrow B=-A$$

$$x: C-2B=0 \rightarrow C=2B=-2A$$

$$1: A-2C=1 \rightarrow A-2(-2A)=1 \rightarrow A=\frac{1}{5}, B=-\frac{1}{5}, C=-\frac{2}{5}.$$

So

$$\frac{1}{(\mathrm{x}-2)(\mathrm{x}^2+1)} = \frac{1}{5}\left(\frac{1}{(\mathrm{x}-2)} - \frac{x+2}{(\mathrm{x}^2+1)}\right).$$

$$\boldsymbol{I} = \int \left(\frac{4}{5}\frac{1}{(\mathrm{x}-2)} + \frac{1}{5}\frac{x+2}{(\mathrm{x}^2+1)}\right)dx = \boldsymbol{I_1} + \boldsymbol{I_2} + c$$

$$\boldsymbol{I_1} = \int \left(\frac{4}{5}\frac{1}{(\mathrm{x}-2)}\right)dx = \frac{4}{5}\int \left(\frac{1}{(\mathrm{x}-2)}\right)dx = \frac{4}{5}\int \frac{d(x-2)}{(x-2)} = \frac{4}{5}\ln(x-2).$$

$$\boldsymbol{I_2} = \int \left(\frac{1}{5}\frac{x+2}{(\mathrm{x}^2+1)}\right)dx = \frac{1}{5}\int \frac{x+2}{\mathrm{x}^2+1}dx = \frac{1}{5}\int \left(\frac{x}{\mathrm{x}^2+1} + \frac{2}{\mathrm{x}^2+1}\right)dx = \boldsymbol{I_3} + \boldsymbol{I_4}$$

$$\boldsymbol{I_3} = \frac{1}{5}\int \left(\frac{x}{\mathrm{x}^2+1}\right)dx = \frac{1}{5}\int \left(\frac{d\left(\frac{x^2}{2}\right)}{\mathrm{x}^2+1}\right) = \frac{1}{10}\int \frac{d(x^2+1)}{x^2+1} = \frac{1}{10}\ln(x^2+1)$$

$$\boldsymbol{I_4} = \frac{1}{5}\int \left(\frac{2}{\mathrm{x}^2+1}\right)dx = \frac{2}{5}\int \frac{dx}{\mathrm{x}^2+1} = \frac{2}{5}\tan^{-1}x.$$

Finally we have

$$\boldsymbol{I} = \frac{4}{5}\ln(x-2) + \frac{1}{10}\ln(x^2+1) + \frac{2}{5}\tan^{-1}x + c.$$

Answer: $\boldsymbol{\frac{4}{5}\ln(x-2) + \frac{1}{10}\ln(x^2+1) + \frac{2}{5}\tan^{-1}x + c}$.