Question:

$$\cot A + \csc A - \frac{1}{\cot A} - \csc A + 1 = 1 + \frac{\cos A}{\sin A}$$

Solution:

Take the left side of the expression and simplify it:

$$\cot A + \csc A - \frac{1}{\cot A} - \csc A + 1 = \cot A - \frac{1}{\cot A} + 1 = \frac{\cos A}{\sin A} - \frac{1}{\frac{\cos A}{\sin A}} + 1 = \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} + 1$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A cos\ A} + 1$$

Substitute this into the initial expression:

$$\frac{\cos^2 A - \sin^2 A}{\sin A cos\ A} + 1 = 1 + \frac{\cos A}{\sin A}$$

$$\frac{\cos^2 A - \sin^2 A}{\sin A cos\ A} = \frac{\cos A}{\sin A}$$

$$\frac{\cos^2 A - \sin^2 A}{\sin A cos\ A} - \frac{\cos A}{\sin A} = 0$$

$$\frac{\cos^2 A - \sin^2 A - \cos^2 A}{\sin A cos\ A} = 0$$

$$\frac{\sin A}{\cos A} = 0$$

$$\sin A = 0$$

But $\sin A$ can't be equal to zero, because $\cot A$ doesn't exist in this case. So our equation has no roots.