



PA=2 cm

PC=11 cm

angle ABC=60°

BP -?

Denote angle ABP as a. Then angle PBC = ABC-ABP=60°-a.

Triangle ABP:

$\sin a = AP/BP$ (from definition), therefore $BP=AP/\sin(60^\circ - a)$ (1)

Triangle PBC:

$\sin(60^\circ - a) = PC/BP$ (from definition), therefore $BP=PC/\sin(60^\circ - a)$ (2)

From (1) and (2):

$AP/\sin a = PC/\sin(60^\circ - a)$ therefore $PC/AP = \sin(60^\circ - a)/\sin a$ (3)

From formula of angle difference for sine ($\sin a - b = \sin a * \cos b - \sin b * \cos a$)

$\sin(60^\circ - a) = \sin 60^\circ * \cos a - \cos 60^\circ * \sin a = \frac{\sqrt{3}}{2} \cos a - \frac{1}{2} \sin a$ (4)

Substitute (4) into (3)

$PC/AP = \frac{\sqrt{3}}{2} (\cos a / \sin a) - \frac{1}{2} (\sin a / \sin a) = \frac{\sqrt{3}}{2} \text{ctg } a - \frac{1}{2} = 11/2$ using elementary math $\text{ctg } a = \frac{10}{\sqrt{3}}$

Triangle ABP:

$\text{ctg } a = AB/AP$

$AB = AP * \text{ctg } a$, using Pythagorean theorem

$BP = \sqrt{AB^2 + AP^2} = \sqrt{\frac{404}{3}}$

Answer: $BP = \sqrt{\frac{404}{3}}$