

Task. Two representatives are chosen at random from a group of 180 students that has 90 girls and 90 boys. What is the probability that the representatives selected are either both boys or both girls?

Choices:

A.) $\frac{90}{180} * \frac{89}{179}$

B.) $\frac{2}{90} + \frac{2}{90}$

C.) $2 * \frac{90}{180} * \frac{89}{179}$

D.) $2 * \frac{2}{90} * \frac{2}{88}$

Proof. Consider the following two events:

$B = \{ \text{both boys are chosen} \}$

$G = \{ \text{both girls are chosen} \}$

We need to find the probability $P(B \cup G)$. Since B and G are mutually exclusive events

$$P(B \cup G) = P(B) + P(G).$$

Assume that a choice of each boy and each girl is independent.

Then the probability to choose the first boy is $90/180$, while the probability to choose The number of choices of 2 boys from 90 is equal to the binomial coefficient

$$C_{90}^2 = \frac{90!}{2!(90-2)!} = \frac{90 * 89}{2},$$

and the number of choices of 2 boys from 180 people is

$$C_{180}^2 = \frac{180!}{2!(180-2)!} = \frac{180 * 179}{2},$$

Then

$$P(B) = \frac{C_{90}^2}{C_{180}^2} = \frac{90 * 89}{2} \frac{2}{180 * 179} = \frac{90}{180} \frac{89}{179}.$$

Evidently, $P(G) = P(B)$ since we also choose two people (two girls) among 90 girls in the group of 180 people. Hence

$$P(B \cup G) = 2 * \frac{90}{180} * \frac{89}{179}.$$

Thus the correct answer is C)