first derivative and second derivative of f(x)= e^(1/x)

The product rule:

$$(f * g)' = g * f' + f * g'$$

The derivative of the function of a function h(x) = f(g(x)) with respect to x is:

$$h(x)' = f'\big(g(x)\big) * g'(x)$$

derivative of exponent:

$$\frac{d}{dx}(e^x) = e^x$$

derivative of x^n:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

So, the first derivative equals:

$$f'(x) = \frac{d}{dx}\left(e^{\frac{1}{x}}\right) = e^{\frac{1}{x}}\frac{d}{dx}\left(\frac{1}{x}\right) = e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right) = -\frac{e^{\frac{1}{x}}}{x^2}$$

and the second derivative:

$$f''(x) = \frac{d}{dx}\left(-\frac{e^{\frac{1}{x}}}{x^2}\right) = -\left(\frac{1}{x^2}\frac{d}{dx}\left(e^{\frac{1}{x}}\right) + e^{\frac{1}{x}}\frac{d}{dx}\left(\frac{1}{x^2}\right)\right) = -\left(-\frac{e^{\frac{1}{x}}}{x^4} - \frac{2e^{\frac{1}{x}}}{x^3}\right) = \frac{e^{\frac{1}{x}}}{x^4}(1 + 2x)$$

Answer: the first derivative $= -\frac{e^{\frac{1}{x}}}{x^2}$, the second derivative $= \frac{e^{\frac{1}{x}}}{x^4}(1 + 2x)$