If $y = e^{ax} cos^3(x) sin^2(x)$ find $\frac{dy}{dx}$.

Solution:

We'll use next rules of differentiation

1) If $y = f(x) \cdot g(x) \cdot r(x)$ then

$$\frac{dy}{dx} = \frac{df(x)}{dx} \cdot g(x) \cdot r(x) + f(x) \cdot \frac{dg(x)}{dx} \cdot r(x) + f(x) \cdot g(x) \cdot \frac{dr(x)}{dx}$$

2) If $y = F(p(x))$ then

$$\frac{dy}{dx} = \frac{dF(p)}{dp} \cdot \frac{dp(x)}{dx}$$

Thus we have

$$\frac{dy}{dx} = \frac{de^{ax}}{dx} cos^3(x) sin^2(x) + e^{ax} \frac{d\left(cos^3(x)\right)}{dx} sin^2(x) + e^{ax} cos^3(x) \frac{d(sin^2(x))}{dx} =$$

$$= ae^{ax} cos^3(x) sin^2(x) - 3e^{ax} cos^2(x) \sin(x)\, sin^2(x) +$$

$$+2e^{ax} cos^3(x) sin(x) cos(x) =$$

$$= e^{ax} cos^2(x) sin(x) \left(a \cdot \cos(x) \sin(x) - 3sin^2(x) + 2cos^2(x)\right) =$$

$$= e^{ax} cos^2(x) sin(x) \left(\frac{a}{2} \sin(2x) - 3(1 - cos^2(x)) + 2cos^2(x)\right) =$$

$$= e^{ax} cos^2(x) sin(x) \left(-3 + \frac{a}{2} \sin(2x) + 5cos^2(x)\right)$$

Answer:

$$\frac{dy}{dx} = e^{ax} cos^2(x) sin(x) \left(-3 + \frac{a}{2} \sin(2x) + 5cos^2(x)\right)$$