We have reviewed and slightly altered the problem because it was incorrect

$$\csc\theta\,(\cot\theta + \csc\theta) + \csc\theta\,(\tan\theta + \sec\theta) = 2\csc\theta\csc 2\theta\,(1 + \sin\theta + \cos\theta)$$

Solution.

Take $\csc\theta$ out of the brackets:

$$\csc\theta\,(\cot\theta + \csc\theta) + \csc\theta\,(\tan\theta + \sec\theta) = \csc\theta\,(\cot\theta + \tan\theta + \csc\theta + \sec\theta)$$

We have

$$\cot\theta + \tan\theta$$

We know that $\cot\theta = \frac{\cos\theta}{\sin\theta}$ *and* $\tan\theta = \frac{\sin\theta}{\cos\theta}$*, then summarize them:*

$$\cot\theta + \tan\theta = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

Use the basic relationship between the sine and the cosine: $\cos^2\theta + \sin^2\theta = 1$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$$

And at the end

$$\frac{2}{\sin 2\theta} = \mathbf{2\csc 2\theta = \cot\theta + \tan\theta}$$

Similarly, for $\csc\theta + \sec\theta$:

$$\csc\theta + \sec\theta = \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$$

Reduce to a common denominator:

$$\frac{1}{\sin\theta} + \frac{1}{\cos\theta} = \frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta} = (\cos\theta + \sin\theta)\frac{2}{2\sin\theta\cos\theta} = \mathbf{2\csc 2\theta\,(\cos\theta + \sin\theta) = \csc\theta + \sec\theta}$$

Then

$$\mathbf{\csc\theta\,(\cot\theta + \tan\theta + \csc\theta + \sec\theta) = 2\csc\theta\csc 2\theta\,(1 + \sin\theta + \cos\theta)}$$

We have identity.