

We have reviewed and slightly altered the problem because it was incorrect as given:

Show that

$$\frac{\tan x + \sec x - 1}{\tan x + \sec x} - 1 = -\frac{\cos x}{1 + \sin x}$$

Solution.

Start with the left side. Decompose into summands:

$$\frac{\tan x + \sec x - 1}{\tan x + \sec x} - 1 = \frac{\tan x + \sec x}{\tan x + \sec x} - \frac{1}{\tan x + \sec x} - 1 = 1 - \frac{1}{\tan x + \sec x} - 1 = -\frac{1}{\tan x + \sec x}$$

We know that $\tan x = \frac{\sin x}{\cos x}$ and $\sec x = \frac{1}{\cos x}$. Then transform the expression:

$$-\frac{1}{\tan x + \sec x} = -\frac{1}{\frac{\sin x}{\cos x} + \frac{1}{\cos x}} = -1 / \left(\frac{\sin x + 1}{\cos x} \right) = -\frac{\cos x}{\sin x + 1}$$

So we have identity.