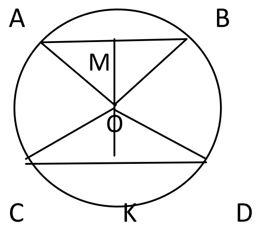
Question #30840

Two parallel chords through a circle, one 16cm and one 30cm, are 23cm apart What is the radius of the circle?

Solution.



Let AB=16, CD=30 and MK=23. Observe that $MK \perp AB$ and $MK \perp CD$. The triangles $\triangle AOB$ and $\triangle COD$ are isosceles, since $AO = BO = OC = OD = R$. It follows that MO and KO are medians of $\triangle AOB$ and $\triangle COD$, respectively and so $AM = MB = \frac{1}{2}AB = 8$ and $CK = KD = \frac{1}{2}CD = 15$. For the right triangles $\triangle AMO$,$\triangle COK$, we have

$$OM = \sqrt{R^2 - AM^2} = \sqrt{R^2 - 8^2},$$

$$OK = \sqrt{R^2 - CK^2} = \sqrt{R^2 - 15^2}.$$

Since $OM + OK = MK = 23$, then $\sqrt{R^2 - 8^2} + \sqrt{R^2 - 15^2} = 23$.

$$R^2 - 64 + 2\sqrt{(R^2 - 64)(R^2 - 225)} + R^2 - 225 = 529,$$

$$2R^2 + 2\sqrt{(R^2 - 64)(R^2 - 225)} = 818,$$

$$\sqrt{(R^2 - 64)(R^2 - 225)} = 409 - R^2,$$

$$R^4 - 289R^2 + 64 \cdot 225 = 409^2 - 2 \cdot 409R^2 + R^4.$$

Thus, we obtain the equation

$$529R^2 = 152881,$$

$$R^2 = 289.$$

Finally, $R = 17$.

Answer. $R = 17$.