**Task.** Find number $s$ of selections of three different numbers from set $A = \{1, 2, 3, ...., 101\}$ so that they form an ariphmetic progression.

**Solution.** We should choose numbers of the form $(a, a+d, a+2d)$ for some $a \in A$ and $d > 0$. In particular, this implies that

$$1 \le a \le a + 2d \le 101.$$

For each $a \in A$ let $s_a$ be the number of $d > 0$ satisfying the inequaly above: $a + 2d \le 101$. Then

$$s = \sum_{a=1}^{101} s_a.$$

Notice that $a + 2d \le 101$ implies that

$$2d \le 101 - a \qquad \Rightarrow \qquad d \le \frac{101 - a}{2}.$$

Thus

$$1 \le d \le \frac{101 - a}{2},$$

and so

$$s_a = \left[\frac{101 - a}{2}\right],$$

where $[x]$ is the integer part of $x$, e.g. $[3.5] = 3$.

Suppose $a = 2k + 1$ is odd. Then

$$s_a = \left[\frac{101 - a}{2}\right] = \left[\frac{101 - 2k - 1}{2}\right] = \left[\frac{100 - 2k}{2}\right] = 50 - k.$$

In particular,

$$\begin{aligned} s_1 &= s_{2*0+1} = 50 - 0 = 50, \\ s_3 &= s_{2*1+1} = 50 - 1 = 49, \\ &\dots\dots\dots\dots\dots\dots \\ s_{99} &= s_{2*49+1} = 50 - 49 = 1, \\ s_{101} &= s_{2*50+1} = 50 - 50 = 0. \end{aligned}$$

Recal that

$$1 + 2 + \cdots + n = \sum_{i=1}^{n} i = \frac{n(n-1)}{2}$$

as the sum of first $n$ members of ariphmetic progression $1, 2, 3, \ldots$

Hence

$$s_{odd} = \sum_{a \le 101,\ a\ is\ odd} s_a = \sum_{n=0}^{50} k = \frac{50 * (50 - 1)}{2} = 1225.$$

Now let $a = 2k$ be even Then

$$s_a = \left[\frac{101 - a}{2}\right] = \left[\frac{101 - 2k}{2}\right] = \left[\frac{101 - 2k}{2}\right] = [50.5 - k] = [50 - k + 0.5] = 50 - k.$$

In particular,

$$\begin{aligned} s_2 &= s_{2*1} = 50 - 1 = 49, \\ s_4 &= s_{2*2} = 50 - 2 = 48, \\ &\dots\dots\dots\dots\dots\dots \\ s_{98} &= s_{2*49} = 50 - 49 = 1, \\ s_{100} &= s_{2*50} = 50 - 50 = 0. \end{aligned}$$

Hence

$$s_{even} = \sum_{a \le 100,\ a\ is\ even} s_a = \sum_{n=0}^{49} k = \frac{49 * (49 - 1)}{2} = 1176.$$

Thus

$$s = s_{odd} + s_{even} = 1225 + 1176 = 2401.$$