Task. Find $f'(x)$ for $f(x) = x^{e^x}$.
Solution. Recall that for any differentiable functions $u(x)$ and $v(x)$ we have that

$$\left(u(x)^{v(x)}\right)' = u(x)^{v(x)} v'(x) \ln u(x) + v(x) \cdot u(x)^{v(x)-1} \cdot u'(x).$$

In our case

$$u(x) = x, \qquad v(x) = e^x,$$

so

$$u'(x) = 1, \qquad v'(x) = e^x.$$

Therefore

$$\left(x^{e^x}\right)' = x^{e^x} (e^x)' \ln x + e^x x^{e^x-1} x' = x^{e^x} e^x \ln x + e^x x^{e^x-1} = x^{e^x-1} e^x (x \ln x + 1).$$