

For the function $f(x) = \frac{e^x}{x^2}$. Find the intervals of increase and decrease. Find the local maximum and minimum points.

Increasing/Decreasing Test Theorem

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) .

1. if $f'(x) > 0$ for all x in (a, b) , then f is increasing on $[a, b]$.

2. if $f'(x) < 0$ for all x in (a, b) , then f is decreasing on $[a, b]$.

$$f'(x) = -\frac{2e^x}{x^3} + \frac{e^x}{x^2} = \frac{e^x}{x^2} \left(1 - \frac{2}{x}\right)$$

$$f'(x) = 0 \text{ or } f'(x) = \infty$$

$$1 - \frac{2}{x} = 0 \text{ therefore } x = 2 \text{ or } x = 0$$

For $x < 0$ $f'(x) > 0$ - interval of increase

For $0 < x < 2$ $f'(x) < 0$ - interval of decrease

For $x > 2$ $f'(x) > 0$ - interval of increase

Local maximum and minimum points at $f'(x) = 0$, therefore $x = 2$

$f(2) = \frac{e^2}{4}$ and obviously $f(2) < f(10)$ therefore $x = 2$ – local minimum.

Answer: $x < 0$ and $x > 2$ - intervals of increase, $0 < x < 2$ - interval of decrease; $x = 2$ – local minimum.