

For the function  $f(x) = \frac{e^x}{x^2}$ . Find the intervals of increase and decrease. Find the local maximum and minimum points.

#### Increasing/Decreasing Test Theorem

let  $f$  be a function that is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ .

1. if  $f'(x) > 0$  for all  $x$  in  $(a, b)$ , then  $f$  is increasing on  $[a, b]$ .

2. if  $f'(x) < 0$  for all  $x$  in  $(a, b)$ , then  $f$  is decreasing on  $[a, b]$ .

$$f'(x) = -\frac{2e^x}{x^3} + \frac{e^x}{x^2} = \frac{e^x}{x^2} \left(1 - \frac{2}{x}\right)$$

$$f'(x) = 0 \text{ or } f'(x) = \infty$$

$$1 - \frac{2}{x} = 0 \text{ therefore } x = 2 \text{ or } x = 0$$

For  $x < 0$   $f'(x) > 0$  - interval of increase

For  $0 < x < 2$   $f'(x) < 0$  - interval of decrease

For  $x > 2$   $f'(x) > 0$  - interval of increase

Local maximum and minimum points at  $f'(x) = 0$ , therefore  $x = 2$

$f(2) = \frac{e^2}{4}$  and obviously  $f(2) < f(10)$  therefore  $x = 2$  - local minimum.

Answer:  $x < 0$  and  $x > 2$  - intervals of increase,  $0 < x < 2$  - interval of decrease;  $x = 2$  - local minimum.