

$$a^x \cdot b^{2y} = c^3$$

and

$$a^{2x} \cdot b^y = c^5.$$

Find x, y.

Solution:

We have next system of equations

$$\begin{cases} a^x \cdot b^{2y} = c^3, \\ a^{2x} \cdot b^y = c^5. \end{cases}$$

Suppose that $a > 0, b > 0$ and $c > 0$. Because

$$\ln(d^k) = k \ln(d)$$

and

$$\ln(d \cdot h) = \ln(d) + \ln(h)$$

then we have

$$\begin{cases} x \ln(a) + 2y \ln(b) = 3 \ln(c), \\ 2x \ln(a) + y \ln(b) = 5 \ln(c). \end{cases}$$

So

$$2(x \ln(a) + 2y \ln(b)) - (2x \ln(a) + y \ln(b)) = 6 \ln(c) - 5 \ln(c),$$

$$3y \ln(b) = \ln(c),$$

$$y = \frac{\ln(c)}{3 \ln(b)} = \frac{\ln(c)}{\ln(b^3)}.$$

Use y we have

$$x \ln(a) + 2 \frac{\ln(c)}{3 \ln(b)} \ln(b) = 3 \ln(c),$$

$$3x \ln(a) + 2 \ln(c) = 9 \ln(c),$$

$$x = \frac{7 \ln(c)}{3 \ln(a)} = \frac{\ln(c^7)}{\ln(a^3)}.$$

Answer:

$$x = \frac{\ln(c^7)}{\ln(a^3)}, y = \frac{\ln(c)}{\ln(b^3)}.$$